Dec. 5th and 6th NOTES/ASSIGNMENT 5.4 Sum and Difference Formulas

$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$

$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$

$\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$\tan(\alpha-\beta)=\dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

Simplify each Expression.

$\cos(25^\circ+15^\circ)$

cos40° 1. cos25°cos15° – sin25°sin15°

$\sin(140-50)^\circ$

sin 90° 3. sin140°cos50° – cos140°sin50°

tan411° 5. $\dfrac{\tan 325^\circ+\tan 86^\circ}{1-\tan 325^\circ\tan 86^\circ}$ 325 86 411

tan80° 7. $\dfrac{\tan 140^\circ-\tan 60^\circ}{1+\tan 140^\circ\tan 60^\circ}$

$\frac{\pi}{7}+\frac{\pi}{5}=$

cos($\frac{12\pi}{35}$) 2. $\cos\frac{\pi}{7}\cos\frac{\pi}{5}-\sin\frac{\pi}{7}\sin\frac{\pi}{5}$

sin1.8 4. sin3cos1.2 – cos3sin1.2 3-1.2=1.8

tan3x 6. $\dfrac{\tan 2x+\tan x}{1-\tan 2x\tan x}$

tan100° 8. $\dfrac{\tan 240^\circ-\tan 140^\circ}{1+\tan 240^\circ\tan 140^\circ}$

Evaluate:

1 9. sin75°cos15° + sin15°cos75° sin90°

$\frac{\sqrt{2}}{2}$ 10. sin15°cos30° + sin30°cos15° sin45

$\frac{\sqrt{2}}{2}$ 11. cos105°cos60° + sin 105°sin60° cos 105 60 45

cute :) 12. cos105°cos15° + sin 105°sin15°

1 13. $\dfrac{\tan 75^\circ-\tan 30^\circ}{1+\tan 75^\circ\tan 30^\circ}$

$\frac{\sqrt{3}}{3}$ 14. $\dfrac{\tan 60^\circ-\tan 30^\circ}{1+\tan 60^\circ\tan 30^\circ}$

$-\frac{\sqrt{3}}{3}$ 15. $\dfrac{\tan 100^\circ+\tan 50^\circ}{1-\tan 100^\circ\tan 50^\circ}$ tan150°

und 16. $\dfrac{\tan 200^\circ+\tan 70^\circ}{1-\tan 200^\circ\tan 70^\circ}$

0 17. $\cos\frac{5\pi}{12}\cos\frac{\pi}{12}-\sin\frac{5\pi}{12}\sin\frac{\pi}{12}$ cos $\frac{\pi}{2}$

1 18. $\cos\frac{7\pi}{6}\cos\frac{5\pi}{6}-\sin\frac{7\pi}{6}\sin\frac{5\pi}{6}$ cos 2π

0 19. $\sin\frac{4\pi}{3}\cos\frac{\pi}{3}-\sin\frac{\pi}{3}\cos\frac{4\pi}{3}$ sin π

$\frac{\sqrt{3}}{2}$ 20. $\sin\frac{2\pi}{3}\cos\frac{\pi}{3}-\sin\frac{\pi}{3}\cos\frac{2\pi}{3}$ sin $\frac{\pi}{3}$

Find the exact value of each expression. (no calculators)

21. cos 75° = cos(30+45)° = cos30°cos45°-sin30°sin45°

$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}-\frac{1}{2}\cdot\frac{\sqrt{2}}{2}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$

22. cos 165° = cos(120+45)° = cos120°cos45°-sin120°sin45°

$= -\frac{1}{2}\cdot\frac{\sqrt{2}}{2}-\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$

$= \frac{-\sqrt{2}-\sqrt{6}}{4}$

23. sin 105° = sin(60+45)° =

sin60°cos45°+cos60°sin45°

$\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}+\frac{1}{2}\cdot\frac{\sqrt{2}}{2}=$

$\frac{\sqrt{6}+\sqrt{2}}{4}$

24. sin 75° = sin(45+30)° =

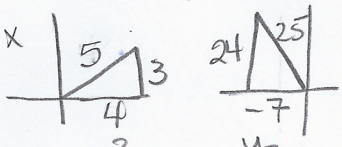
sin45°cos30° + cos45°sin30° =

$\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}+\frac{\sqrt{2}}{2}\cdot\frac{1}{2}=$

$\frac{\sqrt{6}+\sqrt{2}}{4}$

Evaluate.

25. If $\sin x = \frac{3}{5}$ (x is in Quadrant I) and $\sin y = \frac{24}{25}$ (y is in Quadrant II), **find sin (x + y).**



$\sin x = \frac{3}{5}$

$\cos x = \frac{4}{5}$

$\sin y = \frac{24}{25}$

$\cos y = \frac{-7}{25}$

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

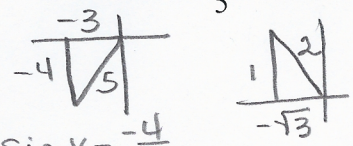
$= \frac{3}{5} \cdot \frac{-7}{25} + \frac{4}{5} \cdot \frac{24}{25}$

$= \frac{-21 + 96}{125}$

$= \frac{75}{125}$

$= \frac{3}{5}$

26. If $\sin x = \frac{-4}{5}$ (x is in Quadrant III) and $\sin y = \frac{1}{2}$ (y is in Quadrant II), **find cos (x – y).**



$\sin x = \frac{-4}{5}$

$\cos x = \frac{-3}{5}$

$\sin y = \frac{1}{2}$

$\cos y = \frac{-\sqrt{3}}{2}$

$\cos(x-y) = \cos x \cos y + \sin x \sin y$

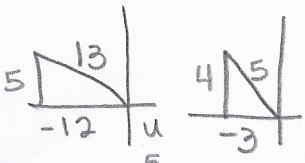
$= \frac{-3}{5} \cdot \frac{-\sqrt{3}}{2} + \frac{-4}{5} \cdot \frac{1}{2}$

$= \frac{3\sqrt{3} - 4}{10}$

Given $\sin u = \frac{5}{13}$ *and* $\cos v = \frac{-3}{5}$**, both in quadrant II. find:**

$\tan u = \frac{5}{-12}$ $\tan v = \frac{4}{-3}$

27. sin (u + v)



$\sin u = \frac{5}{13}$ $\sin v = \frac{4}{5}$

$\cos u = \frac{-12}{13}$ $\cos v = \frac{-3}{5}$

$\sin u \cos v + \cos u \sin v =$

$\frac{5}{13} \cdot \frac{-3}{5} + \frac{-12}{13} \cdot \frac{4}{5} = \frac{-15-48}{65} = -\frac{63}{65}$

28. cos (u + v)

$\cos u \cos v - \sin u \sin v$

$= \frac{-12}{13} \cdot \frac{-3}{5} - \frac{5}{13} \cdot \frac{4}{5}$

$= \frac{36 - 20}{65}$

$= \frac{16}{65}$

29. tan (u + v) =

$\frac{\tan u + \tan v}{1 - \tan u \tan v} =$

$\frac{\frac{-5}{12} + \frac{-4}{3}}{1 - (\frac{-5}{12} \cdot \frac{-4}{3})} =$

$\frac{\frac{-5}{12} + \frac{-4 \cdot 4}{3 \cdot 4}}{\frac{36}{36} - \frac{20}{36}} = \frac{-21}{\cancel{12}} \cdot \frac{\cancel{36}^{3}}{16} = \frac{-63}{16}$

30. sin (u – v)

$\sin u \cos v - \cos u \sin v =$

$\frac{5}{13} \cdot \frac{-3}{5} - \frac{-12}{13} \cdot \frac{4}{5} =$

$\frac{-15}{65} + \frac{48}{65} = \frac{33}{65}$

31. cos (u – v)

$\cos u \cos v + \sin u \sin v =$

$\frac{-12}{13} \cdot \frac{-3}{5} + \frac{5}{13} \cdot \frac{4}{5} =$

$\frac{36}{65} + \frac{20}{65} = \frac{56}{65}$

32. tan (u – v)

$\frac{\tan u - \tan v}{1 + \tan u \tan v} =$

$\frac{\frac{-5}{12} + \frac{+4}{3} \cdot \frac{4}{4}}{1 + \frac{-5}{12} \cdot \frac{-4}{3}} =$

$\frac{\frac{11}{12}}{\frac{36+20}{36}} = \frac{11}{\cancel{12}} \cdot \frac{\cancel{36}^{3}}{56} = \frac{33}{56}$