Dec. 3rd/4th – ASSIGNMENT **5.3 Solving Trig Equations** Solve each equation.

I Collecting Like Terms

1. $5\cos x - 3 = 3\cos x - 4$

$2\cos x = -1$

$\cos x = -\frac{1}{2}$ → cosine neg in QII+III

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

2. $\sin x - \sqrt{3} = 3\sin x$

$-2\sin x = \sqrt{3}$

$\sin x = -\frac{\sqrt{3}}{2}$ → sine negative in QIII+IV

$x = \frac{4\pi}{3}, \frac{5\pi}{3}$

3. $2\cos x + \sqrt{3} = 0$

$2\cos x = -\sqrt{3}$

$\cos x = -\frac{\sqrt{3}}{2}$

$x = \frac{5\pi}{6}, \frac{7\pi}{6}$

4. $4\cos x + 2 = \cos x + 1$

$3\cos x = -1$

$\cos x = -\frac{1}{3}$

$\cos^{-1}(-\frac{1}{3}) = x$

5. $\sqrt{2}\sec x = 2$

$\sec x = \frac{2}{\sqrt{2}}$

$\cos x = \frac{\sqrt{2}}{2}$ ← cosine is positive in QI+IV

$x = \frac{\pi}{4}, \frac{7\pi}{4}$

II Extracting Square Roots

6. $3\sec^2 x = 4$

$\sec^2 x = \frac{4}{3}$

$\sec x = \pm\frac{2}{\sqrt{3}}$

$\cos x = \pm\frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

7. $3\cos^2 x = 3 - \cos^2 x$

$4\cos^2 x = 3$

$\cos^2 x = \frac{3}{4}$

$\cos x = \pm\frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

III Factoring out GCF

8. $\csc^4 x - 4\csc^2 x = 0$

$\csc^2 x(\csc^2 x - 4) = 0$

$\csc^2 x = 0$ $\csc^2 x - 4 = 0$

$\csc x$ never $= 0$ $\csc x = \pm\sqrt{4}$

$\csc x = \pm 2$

$\sin x = \pm\frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

9. $\sin^2 x - \sin x = 0$

$\sin x(\sin x - 1) = 0$

$\sin x = 0$ $\sin x - 1 = 0$

$x = 0, \pi$ $\sin x = 1$

$x = \frac{\pi}{2}$

10. $2\cos^2 x = \cos x$

$2\cos^2 x - \cos x = 0$

$\cos x(2\cos x - 1) = 0$

$\cos x = 0$ $2\cos x - 1 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

11. $\sqrt{3}\tan x \sin x = \sin x$

$\sqrt{3}\tan x \sin x - \sin x = 0$

$\sin x(\sqrt{3}\tan x - 1) = 0$

$\sin x = 0$ $\sqrt{3}\tan x = 1$

$x = 0, \pi$ $\tan x = \frac{1}{\sqrt{3}}$

$x = \frac{\pi}{6}, \frac{7\pi}{6}$

12. $\sqrt{2}\sin x \cos x - 2\sin x = 0$

$\sin x(\sqrt{2}\cos x - 2) = 0$

$\sin x = 0$ $\sqrt{2}\cos x = 2$

$x = 0, \pi$ $\cos x = \frac{2}{\sqrt{2}}$

$x = \frac{\pi}{4}, \frac{7\pi}{4}$

13. $\tan x \sec x = \tan x$

$\tan x \sec x - \tan x = 0$

$\tan x(\sec x - 1) = 0$

$\tan x = 0$ $\sec x = 1$

$x = 0, \pi$ $x = 0$

IV Factoring an Equation of Quadratic Type

14. $2\sin^2 x - 5\sin x = 3$

$2\sin^2 x - 5\sin x - 3 = 0$

$(2\sin x + 1)(\sin x - 3) = 0$

$2\sin x + 1 = 0$ $\quad$ $\sin x - 3 = 0$

$\sin x = -\frac{1}{2}$ $\quad$ $\sin x = 3$ ← NEVER

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

15. $\sec^2 x - \sec x - 2 = 0$

$(\sec x - 2)(\sec x + 1) = 0$

$\sec x = 2$ $\quad$ $\sec x = -1$

$\cos x = \frac{1}{2}$ $\quad$ $\cos x = -1$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$ $\quad$ $x = \pi$

16. $\tan^2 x - 2\tan x + 1 = 0$

$(\tan x - 1)(\tan x - 1) = 0$

$\tan x = 1$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

17. $\sec^2 x + 3\sec x = -2$

$\sec^2 x + 3\sec x + 2 = 0$

$(\sec x + 1)(\sec x + 2) = 0$

$\sec x = -1$ $\quad$ $\sec x = -2$

$x = \pi$ $\quad$ $\cos x = -\frac{1}{2}$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

V. Functions of Multiple Angles

$2x = \frac{3\pi}{2}, \frac{7\pi}{2}$

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

18. $\tan^2 3x = 3$

$\tan 3x = \pm\sqrt{3}$

$3x = \frac{\pi}{3}, \frac{4\pi}{3}, \frac{7\pi}{3}, \frac{10\pi}{3}, \frac{13\pi}{3}, \frac{16\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{3}, \frac{8\pi}{3}, \frac{11\pi}{3}, \frac{14\pi}{3}, \frac{17\pi}{3}$

$x = \frac{\pi}{9}, \frac{4\pi}{9}, \frac{7\pi}{9}, \frac{10\pi}{9}, \frac{13\pi}{9}, \frac{16\pi}{3}, \frac{2\pi}{9}, \frac{5\pi}{9}, \frac{8\pi}{9}, \frac{11\pi}{9}, \frac{14\pi}{9}, \frac{17\pi}{9}$

19. $\sin 2x = -1$

20. $\sec 2x = -2$ →

$\cos 2x = -\frac{1}{2}$

$2x = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

VI Substituting Using Identity Formulas:

21. $2\sec^2 x - 2\tan^2 - 4 = 0$

$2(\sec^2 x - \tan^2 x) - 4 = 0$

$2 - 4 = 0$

$2(\tan^2 x + 1) - 2\tan^2 x - 4 = 0$

no solution

22. $2\cos^2 x - 2\sin^2 x = 1$

$2(1 - \sin^2 x) - 2\sin^2 x = 1$

$2 - 2\sin^2 x - 2\sin^2 x - 1 = 0$

$-4\sin^2 x + 1 = 0$

$\sin^2 x = \frac{1}{4}$

$\sin x = \pm\frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

23. $\sin^2 x - \cos^2 x = 1$

$\sin^2 x - (1 - \sin^2 x) - 1 = 0$

$\sin^2 x - 1 + \sin^2 x - 1 = 0$

$2\sin^2 x - 2 = 0$

$2(\sin^2 x - 1) = 0$

$\sin^2 x - 1 = 0$

$(\sin x + 1)(\sin x - 1) = 0$

$\sin x = -1$ $\quad$ $\sin x = 1$

$x = \frac{3\pi}{2}$ $\quad$ $x = \frac{\pi}{2}$

24. $2\sin^2 x = 2 + \cos x$

$2(1 - \cos^2 x) - 2 - \cos x = 0$

$\cancel{2} - 2\cos^2 x - \cancel{2} - \cos x = 0$

$2\cos^2 x + \cos x = 0$

$\cos x(2\cos x + 1) = 0$

$\cos x = 0$ $\quad$ $\cos x = -\frac{1}{2}$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad$ $x = \frac{2\pi}{3}, \frac{4\pi}{3}$

25. $2\sec^2 x + \tan^2 x = 3$

$2(\tan^2 x + 1) + \tan^2 x - 3 = 0$

$3\tan^2 x + 2 - 3 = 0$

$3\tan^2 x - 1 = 0$

$\tan^2 x = \frac{1}{3}$

$\tan x = \pm\frac{1}{\sqrt{3}}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$