

I. Collecting Like Terms

Restrict domain: $[0, 2\pi)$

$$\begin{aligned} 1. \sin x + \sqrt{2} &= -\sin x \\ 2\sin x + \sqrt{2} &= 0 \\ 2\sin x &= -\sqrt{2} \\ \sin x &= -\frac{\sqrt{2}}{2} \end{aligned}$$

$$\text{III} \quad X = \frac{5\pi}{4}, \frac{7\pi}{4}$$

II. Extracting Square Roots

$$\begin{aligned} 3. 3\tan^2 x - 1 &= 0 \\ 3\tan^2 x &= 1 \\ \tan^2 x &= \frac{1}{3} \\ \sqrt{\tan^2 x} &= \pm \frac{1}{\sqrt{3}} \\ \tan x &= \pm \frac{1}{\sqrt{3}} \\ X &= \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \end{aligned}$$

IV. Factoring an Equation of Quadratic Type

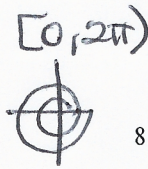
Let $a = \sin x$

$$\begin{aligned} 5. 2\sin^2 x - \sin x - 1 &= 0 \\ 2a^2 - a - 1 &= 0 \\ (2a + 1)(a - 1) &= 0 \\ 2a + 1 = 0 \quad a - 1 = 0 \\ 2a = -1 \quad a = 1 \\ a = -\frac{1}{2} \quad \sin x = 1 \\ \sin x = -\frac{1}{2} \quad X = \frac{7\pi}{6}, \frac{11\pi}{6} \\ X = \frac{\pi}{2} \end{aligned}$$

III

V. Functions of Multiple Angles

$$\begin{aligned} 7. \cos 3x &= 1 \quad \text{3x around} \\ 3x &= 0, 2\pi, 4\pi \\ X &= 0, \frac{2\pi}{3}, \frac{4\pi}{3} \end{aligned}$$



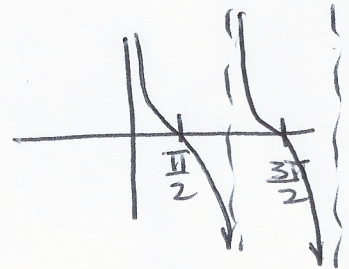
VI. Substituting Using Identity Formulas

$$\begin{aligned} 9. 2\sin^2 x + 3\cos x &= 3 \\ 2(1 - \cos^2 x) + 3\cos x - 3 &= 0 \\ 2 - 2\cos^2 x + 3\cos x - 3 &= 0 \\ -2\cos^2 x + 3\cos x - 1 &= 0 \\ 2\cos^2 x - 3\cos x + 1 &= 0 \\ (2\cos x - 1)(\cos x - 1) &= 0 \\ 2\cos x - 1 = 0 \quad \cos x - 1 = 0 \\ \cos x = \frac{1}{2} \quad \cos x = 1 \\ X = \frac{\pi}{3}, \frac{5\pi}{3} \quad X = 0 \end{aligned}$$

$$\begin{aligned} 2. \sqrt{3}\sec x + 2 &= 0 \\ \sqrt{3}\sec x &= -2 \\ \sec x &= \frac{-2}{\sqrt{3}} \\ \cos x &= -\frac{\sqrt{3}}{2} \\ \text{II+III} \quad X &= \frac{5\pi}{6}, \frac{7\pi}{6} \end{aligned}$$

III. Factoring out a GCF

$$\begin{aligned} 4. \cot x \cos^2 x &= \cot x \\ \cot x \cos^2 x - \cot x &= 0 \\ \cot x (\cos^2 x - 1) &= 0 \\ \cot x (\cos x + 1)(\cos x - 1) &= 0 \\ \cot x = 0 \quad \cos x + 1 = 0 \quad \cos x - 1 = 0 \\ X = \frac{\pi}{2}, \frac{3\pi}{2} \quad \cos x = -1 \quad \cos x = 1 \\ \quad \quad \quad \pi \quad \quad \quad 0 \end{aligned}$$



$$\begin{aligned} 6. 2\cos^2 x + 3\cos x + 1 &= 0 \\ (2\cos x + 1)(\cos x + 1) &= 0 \\ 2\cos x + 1 = 0 \quad \cos x + 1 = 0 \\ 2\cos x = -1 \quad \cos x = -1 \\ \cos x = -\frac{1}{2} \quad \cos x = -1 \\ X = \frac{2\pi}{3}, \frac{4\pi}{3} \quad X = \pi \end{aligned}$$

$$\begin{aligned} 8. 2\sin^2 2x &= 1 \quad \text{2x times around} \\ \sin^2 2x &= \frac{1}{2} \\ \sin 2x &= \pm \frac{1}{\sqrt{2}} = \pm \frac{\sqrt{2}}{2} \\ 2x &= \left\{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \right\} \\ &= \left\{ \frac{9\pi}{4}, \frac{11\pi}{4}, \frac{13\pi}{4}, \frac{15\pi}{4} \right\} \\ X &= \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8} \end{aligned}$$