333202_0905.qxd 12/5/05 11:37 AM Page 683

9.5 The Binomial Theorem

What you should learn

- Use the Binomial Theorem to calculate binomial coefficients.
- Use Pascal's Triangle to calculate binomial coefficients.
- Use binomial coefficients to write binomial expansions.

Why you should learn it

You can use binomial coefficients to model and solve real-life problems. For instance, in Exercise 80 on page 690, you will use binomial coefficients to write the expansion of a model that represents the amounts of child support collected in the U. S.

© Vince Streano/Corbis

Binomial Coefficients

Recall that a *binomial* is a polynomial that has two terms. In this section, you will study a formula that gives a quick method of raising a binomial to a power. To begin, look at the expansion of $(x + y)^n$ for several values of n.

$$(x + y)^0 = 1$$

$$(x + y)^1 = x + y$$

$$(x + y)^2 = x^2 + 2xy + y^2$$

$$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

$$(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$

$$(x + y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$$

There are several observations you can make about these expansions.

1. In each expansion, there are $n + 1$ terms.
2. In each expansion, x and y have symmetrical roles. The powers of x decrease by 1 in successive terms, whereas the powers of y increase by 1.
3. The sum of the powers of each term is n. For instance, in the expansion of $(x + y)^5$, the sum of the powers of each term is 5.

$$(x + y)^5 = x^5 + 5\overbrace{x^4y^1}^{4+1=5} + 10\overbrace{x^3y^2}^{3+2=5} + 10x^2y^3 + 5x^1y^4 + y^5$$

4. The coefficients increase and then decrease in a symmetric pattern.

The coefficients of a binomial expansion are called **binomial coefficients.** To find them, you can use the **Binomial Theorem.**

The Binomial Theorem

In the expansion of $(x + y)^n$

$$(x + y)^n = x^n + nx^{n-1}y + \cdots + {}_nC_r\, x^{n-1}y^r + \cdots + nxy^{n-1} + y^n$$

the coefficient of $x^{n-r}y^r$ is

$${}_nC_r = \frac{n!}{(n-r)!r!}.$$

The symbol $\binom{n}{r}$ is often used in place of ${}_nC_r$ to denote binomial coefficients.

For a proof of the Binomial Theorem, see Proofs in Mathematics on page 724.

Technology

Most graphing calculators are programmed to evaluate ${}_nC_r$. Consult the user's guide for your calculator and then evaluate ${}_8C_5$. You should get an answer of 56.

Example 1 Finding Binomial Coefficients

Find each binomial coefficient.

a. ${}_8C_2$ **b.** $\binom{10}{3}$ **c.** ${}_7C_0$ **d.** $\binom{8}{8}$

Solution

a. $${}_8C_2 = \frac{8!}{6! \cdot 2!} = \frac{(8 \cdot 7) \cdot \cancel{6!}}{\cancel{6!} \cdot 2!} = \frac{8 \cdot 7}{2 \cdot 1} = 28$$

b. $$\binom{10}{3} = \frac{10!}{7! \cdot 3!} = \frac{(10 \cdot 9 \cdot 8) \cdot \cancel{7!}}{\cancel{7!} \cdot 3!} = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1} = 120$$

c. $${}_7C_0 = \frac{\cancel{7!}}{\cancel{7!} \cdot 0!} = 1$$ **d.** $$\binom{8}{8} = \frac{\cancel{8!}}{0! \cdot \cancel{8!}} = 1$$

CHECKPOINT Now try Exercise 1.

When $r \neq 0$ and $r \neq n$, as in parts (a) and (b) above, there is a simple pattern for evaluating binomial coefficients that works because there will always be factorial terms that divide out from the expression.

$${}_8C_2 = \frac{\overbrace{8 \cdot 7}^{\text{2 factors}}}{\underbrace{2 \cdot 1}_{\text{2 factors}}} \quad \text{and} \quad \binom{10}{3} = \frac{\overbrace{10 \cdot 9 \cdot 8}^{\text{3 factors}}}{\underbrace{3 \cdot 2 \cdot 1}_{\text{3 factors}}}$$

Example 2 Finding Binomial Coefficients

Find each binomial coefficient.

a. ${}_7C_3$ **b.** $\binom{7}{4}$ **c.** ${}_{12}C_1$ **d.** $\binom{12}{11}$

Solution

a. $${}_7C_3 = \frac{7 \cdot \cancel{6} \cdot 5}{\cancel{3} \cdot \cancel{2} \cdot 1} = 35$$

b. $$\binom{7}{4} = \frac{7 \cdot \cancel{6} \cdot 5 \cdot \cancel{4}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1} = 35$$

c. $${}_{12}C_1 = \frac{12}{1} = 12$$

d. $$\binom{12}{11} = \frac{12!}{1! \cdot 11!} = \frac{(12) \cdot \cancel{11!}}{1! \cdot \cancel{11!}} = \frac{12}{1} = 12$$

CHECKPOINT Now try Exercise 7.

It is not a coincidence that the results in parts (a) and (b) of Example 2 are the same and that the results in parts (c) and (d) are the same. In general, it is true that

$${}_nC_r = {}_nC_{n-r}.$$

This shows the symmetric property of binomial coefficients that was identified earlier.

333202_0905.qxd 12/5/05 11:37 AM Page 685

Pascal's Triangle

There is a convenient way to remember the pattern for binomial coefficients. By arranging the coefficients in a triangular pattern, you obtain the following array, which is called **Pascal's Triangle.** This triangle is named after the famous French mathematician Blaise Pascal (1623–1662).

$$
\begin{array}{ccccccccccccccc}
 & & & & & & & 1 & & & & & & & \\
 & & & & & & 1 & & 1 & & & & & & \\
 & & & & & 1 & & 2 & & 1 & & & & & \\
 & & & & 1 & & 3 & & 3 & & 1 & & & & \\
 & & & 1 & & 4 & & 6 & & 4 & & 1 & & & \\
 & & 1 & & 5 & & 10 & & 10 & & 5 & & 1 & & \quad 4+6=10 \\
 & 1 & & 6 & & 15 & & 20 & & 15 & & 6 & & 1 & \\
1 & & 7 & & 21 & & 35 & & 35 & & 21 & & 7 & & 1 \quad 15+6=21
\end{array}
$$

The first and last numbers in each row of Pascal's Triangle are 1. Every other number in each row is formed by adding the two numbers immediately above the number. Pascal noticed that numbers in this triangle are precisely the same numbers that are the coefficients of binomial expansions, as follows.

$$
\begin{array}{rcll}
(x+y)^0 &=& 1 & \text{0th row} \\
(x+y)^1 &=& 1x + 1y & \text{1st row} \\
(x+y)^2 &=& 1x^2 + 2xy + 1y^2 & \text{2nd row} \\
(x+y)^3 &=& 1x^3 + 3x^2y + 3xy^2 + 1y^3 & \text{3rd row} \\
(x+y)^4 &=& 1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4 & \vdots \\
(x+y)^5 &=& 1x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + 1y^5 & \\
(x+y)^6 &=& 1x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + 1y^6 & \\
(x+y)^7 &=& 1x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + 1y^7 &
\end{array}
$$

The top row in Pascal's Triangle is called the *zeroth row* because it corresponds to the binomial expansion $(x + y)^0 = 1$. Similarly, the next row is called the *first row* because it corresponds to the binomial expansion $(x + y)^1 = 1(x) + 1(y)$. In general, the *nth row* in Pascal's Triangle gives the coefficients of $(x + y)^n$.

Exploration

Complete the table and describe the result.

n	r	${}_nC_r$	${}_nC_{n-r}$
9	5		
7	1		
12	4		
6	0		
10	7		

What characteristic of Pascal's Triangle is illustrated by this table?

Example 3 Using Pascal's Triangle

Use the seventh row of Pascal's Triangle to find the binomial coefficients.

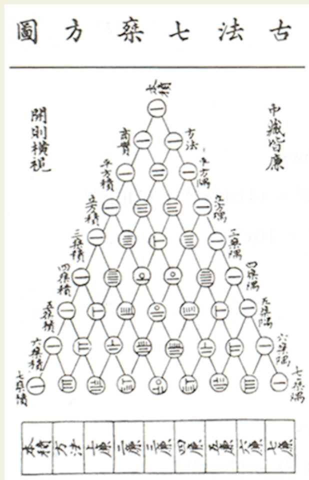
${}_8C_0, {}_8C_1, {}_8C_2, {}_8C_3, {}_8C_4, {}_8C_5, {}_8C_6, {}_8C_7, {}_8C_8$

Solution

$$
\begin{array}{ccccccccccccccccc}
 & 1 & & 7 & & 21 & & 35 & & 35 & & 21 & & 7 & & 1 & \\
1 & & 8 & & 28 & & 56 & & 70 & & 56 & & 28 & & 8 & & 1 \\
{}_8C_0 & & {}_8C_1 & & {}_8C_2 & & {}_8C_3 & & {}_8C_4 & & {}_8C_5 & & {}_8C_6 & & {}_8C_7 & & {}_8C_8
\end{array}
$$

CHECKPOINT Now try Exercise 11.

333202_0905.qxd 12/5/05 11:37 AM Page 686



Historical Note
Precious Mirror "Pascal's" Triangle and forms of the Binomial Theorem were known in Eastern cultures prior to the Western "discovery" of the theorem. A Chinese text entitled *Precious Mirror* contains a triangle of binomial expansions through the eighth power.

Binomial Expansions

As mentioned at the beginning of this section, when you write out the coefficients for a binomial that is raised to a power, you are **expanding a binomial.** The formulas for binomial coefficients give you an easy way to expand binomials, as demonstrated in the next four examples.

Example 4 Expanding a Binomial

Write the expansion for the expression

$(x + 1)^3$.

Solution

The binomial coefficients from the third row of Pascal's Triangle are

1, 3, 3, 1.

So, the expansion is as follows.

$$(x + 1)^3 = (1)x^3 + (3)x^2(1) + (3)x(1^2) + (1)(1^3)$$
$$= x^3 + 3x^2 + 3x + 1$$

CHECKPOINT Now try Exercise 15.

To expand binomials representing *differences* rather than sums, you alternate signs. Here are two examples.

$$(x - 1)^3 = x^3 - 3x^2 + 3x - 1$$
$$(x - 1)^4 = x^4 - 4x^3 + 6x^2 - 4x + 1$$

Example 5 Expanding a Binomial

Write the expansion for each expression.

a. $(2x - 3)^4$ **b.** $(x - 2y)^4$

Solution

The binomial coefficients from the fourth row of Pascal's Triangle are

1, 4, 6, 4, 1.

Therefore, the expansions are as follows.

a. $(2x - 3)^4 = (1)(2x)^4 - (4)(2x)^3(3) + (6)(2x)^2(3^2) - (4)(2x)(3^3) + (1)(3^4)$

$= 16x^4 - 96x^3 + 216x^2 - 216x + 81$

b. $(x - 2y)^4 = (1)x^4 - (4)x^3(2y) + (6)x^2(2y)^2 - (4)x(2y)^3 + (1)(2y)^4$

$= x^4 - 8x^3y + 24x^2y^2 - 32xy^3 + 16y^4$

CHECKPOINT Now try Exercise 19.

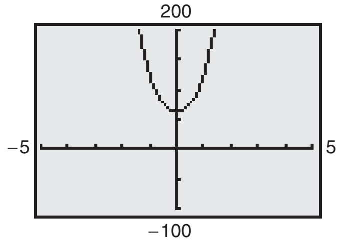
Group Activity

Add the binomial coefficients in each of the first five rows of Pascal's Triangle. What pattern do you see? Work together to use the pattern to find the sums of the terms in the 10th, 15th, and 20th rows of Pascal's Triangle. Check your answer by actually adding the terms of the 10th, 15th, and 20th rows.

333202_0905.qxd 12/5/05 11:37 AM Page 687

Technology

You can use a graphing utility to check the expansion in Example 6. Graph the original binomial expression and the expansion in the same viewing window. The graphs should coincide as shown below.



Activities

1. Evaluate ${}_8C_3$.

   Answer: $\dfrac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 56$

2. Expand and simplify $(x - 3)^4$.

   Answer: $x^4 - 12x^3 + 54x^2 - 108x + 81$

3. Find the fifth term in the expansion of $(2x + 1)^9$.

   Answer: ${}_9C_4(2x)^{9-4}(1)^4 = 4032x^5$

Example 6 Expanding a Binomial

Write the expansion for $(x^2 + 4)^3$.

Solution

Use the third row of Pascal's Triangle, as follows.

$$(x^2 + 4)^3 = (1)(x^2)^3 + (3)(x^2)^2(4) + (3)x^2(4^2) + (1)(4^3)$$
$$= x^6 + 12x^4 + 48x^2 + 64$$

CHECKPOINT Now try Exercise 29.

Sometimes you will need to find a specific term in a binomial expansion. Instead of writing out the entire expansion, you can use the fact that, from the Binomial Theorem, the $(r + 1)$th term is ${}_nC_r\,x^{n-r}y^r$.

Example 7 Finding a Term in a Binomial Expansion

a. Find the sixth term of $(a + 2b)^8$.

b. Find the coefficient of the term a^6b^5 in the expansion of $(3a - 2b)^{11}$.

Solution

a. Remember that the formula is for the $(r + 1)$th term, so r is one less than the number of the term you are looking for. So, to find the sixth term in this binomial expansion, use $r = 5$, $n = 8$, $x = a$, and $y = 2b$, as shown.

$${}_8C_5a^{8-5}(2b)^5 = 56 \cdot a^3 \cdot (2b)^5 = 56(2^5)a^3b^5 = 1792a^3b^5.$$

b. In this case, $n = 11$, $r = 5$, $x = 3a$, and $y = -2b$. Substitute these values to obtain

$${}_nC_r\,x^{n-r}y^r = {}_{11}C_5(3a)^6(-2b)^5$$
$$= (462)(729a^6)(-32b^5)$$
$$= -10{,}777{,}536a^6b^5.$$

So, the coefficient is $-10{,}777{,}536$.

CHECKPOINT Now try Exercise 41.

WRITING ABOUT MATHEMATICS

Error Analysis You are a math instructor and receive the following solutions from one of your students on a quiz. Find the error(s) in each solution. Discuss ways that your student could avoid the error(s) in the future.

a. Find the second term in the expansion of $(2x - 3y)^5$.

$$\cancel{5(2x)^4(3y)^2 = 720x^4y^2}$$

b. Find the fourth term in the expansion of $\left(\frac{1}{2}x + 7y\right)^6$.

$$\cancel{{}_6C_4\left(\tfrac{1}{2}x\right)^2(7y)^4 = 9003.75x^2y^4}$$

9.5 Exercises

VOCABULARY CHECK: Fill in the blanks.

1. The coefficients of a binomial expansion are called ________ ________.
2. To find binomial coefficients, you can use the ________ ________ or ________ ________.
3. The notation used to denote a binomial coefficient is ________ or ________.
4. When you write out the coefficients for a binomial that is raised to a power, you are ________ a ________.

PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

In Exercises 1–10, calculate the binomial coefficient.

1. ${}_5C_3$ **2.** ${}_8C_6$

3. ${}_{12}C_0$ **4.** ${}_{20}C_{20}$

5. ${}_{20}C_{15}$ **6.** ${}_{12}C_5$

7. $\binom{10}{4}$ **8.** $\binom{10}{6}$

9. $\binom{100}{98}$ **10.** $\binom{100}{2}$

In Exercises 11–14, evaluate using Pascal's Triangle.

11. $\binom{8}{5}$ **12.** $\binom{8}{7}$

13. ${}_7C_4$ **14.** ${}_6C_3$

In Exercises 15–34, use the Binomial Theorem to expand and simplify the expression.

15. $(x + 1)^4$ **16.** $(x + 1)^6$

17. $(a + 6)^4$ **18.** $(a + 5)^5$

19. $(y - 4)^3$ **20.** $(y - 2)^5$

21. $(x + y)^5$ **22.** $(c + d)^3$

23. $(r + 3s)^6$ **24.** $(x + 2y)^4$

25. $(3a - 4b)^5$ **26.** $(2x - 5y)^5$

27. $(2x + y)^3$ **28.** $(7a + b)^3$

29. $(x^2 + y^2)^4$ **30.** $(x^2 + y^2)^6$

31. $\left(\frac{1}{x} + y\right)^5$ **32.** $\left(\frac{1}{x} + 2y\right)^6$

33. $2(x - 3)^4 + 5(x - 3)^2$

34. $3(x + 1)^5 - 4(x + 1)^3$

In Exercises 35–38, expand the binomial by using Pascal's Triangle to determine the coefficients.

35. $(2t - s)^5$ **36.** $(3 - 2z)^4$

37. $(x + 2y)^5$ **38.** $(2v + 3)^6$

In Exercises 39–46, find the specified *n*th term in the expansion of the binomial.

39. $(x + y)^{10}, \quad n = 4$ **40.** $(x - y)^6, \quad n = 7$

41. $(x - 6y)^5, \quad n = 3$ **42.** $(x - 10z)^7, \quad n = 4$

43. $(4x + 3y)^9, \quad n = 8$ **44.** $(5a + 6b)^5, \quad n = 5$

45. $(10x - 3y)^{12}, \quad n = 9$ **46.** $(7x + 2y)^{15}, \quad n = 7$

In Exercises 47–54, find the coefficient *a* of the term in the expansion of the binomial.

	Binomial	*Term*
47.	$(x + 3)^{12}$	ax^5
48.	$(x^2 + 3)^{12}$	ax^8
49.	$(x - 2y)^{10}$	ax^8y^2
50.	$(4x - y)^{10}$	ax^2y^8
51.	$(3x - 2y)^9$	ax^4y^5
52.	$(2x - 3y)^8$	ax^6y^2
53.	$(x^2 + y)^{10}$	ax^8y^6
54.	$(z^2 - t)^{10}$	az^4t^8

In Exercises 55–58, use the Binomial Theorem to expand and simplify the expression.

55. $\left(\sqrt{x} + 3\right)^4$

56. $\left(2\sqrt{t} - 1\right)^3$

57. $(x^{2/3} - y^{1/3})^3$

58. $(u^{3/5} + 2)^5$

In Exercises 59–62, expand the expression in the difference quotient and simplify.

$$\frac{f(x + h) - f(x)}{h} \qquad \text{Difference quotient}$$

59. $f(x) = x^3$ **60.** $f(x) = x^4$

61. $f(x) = \sqrt{x}$ **62.** $f(x) = \frac{1}{x}$

333202_0905.qxd 12/5/05 11:37 AM Page 689

In Exercises 63–68, use the Binomial Theorem to expand the complex number. Simplify your result.

63. $(1 + i)^4$

64. $(2 - i)^5$

65. $(2 - 3i)^6$

66. $(5 + \sqrt{-9})^3$

67. $\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^3$

68. $(5 - \sqrt{3}i)^4$

Approximation **In Exercises 69–72, use the Binomial Theorem to approximate the quantity accurate to three decimal places. For example, in Exercise 69, use the expansion**

$$(1.02)^8 = (1 + 0.02)^8 = 1 + 8(0.02) + 28(0.02)^2 + \cdots.$$

69. $(1.02)^8$

70. $(2.005)^{10}$

71. $(2.99)^{12}$

72. $(1.98)^9$

Graphical Reasoning **In Exercises 73 and 74, use a graphing utility to graph f and g in the same viewing window. What is the relationship between the two graphs? Use the Binomial Theorem to write the polynomial function g in standard form.**

73. $f(x) = x^3 - 4x, \quad g(x) = f(x + 4)$

74. $f(x) = -x^4 + 4x^2 - 1, \quad g(x) = f(x - 3)$

Probability **In Exercises 75–78, consider n independent trials of an experiment in which each trial has two possible outcomes: "success" or "failure." The probability of a success on each trial is p, and the probability of a failure is $q = 1 - p$. In this context, the term ${}_nC_k\, p^k q^{n-k}$ in the expansion of $(p + q)^n$ gives the probability of k successes in the n trials of the experiment.**

75. A fair coin is tossed seven times. To find the probability of obtaining four heads, evaluate the term

$${}_7C_4\left(\tfrac{1}{2}\right)^4\left(\tfrac{1}{2}\right)^3$$

in the expansion of $\left(\frac{1}{2} + \frac{1}{2}\right)^7$.

76. The probability of a baseball player getting a hit during any given time at bat is $\frac{1}{4}$. To find the probability that the player gets three hits during the next 10 times at bat, evaluate the term

$${}_{10}C_3\left(\tfrac{1}{4}\right)^3\left(\tfrac{3}{4}\right)^7$$

in the expansion of $\left(\frac{1}{4} + \frac{3}{4}\right)^{10}$.

77. The probability of a sales representative making a sale with any one customer is $\frac{1}{3}$. The sales representative makes eight contacts a day. To find the probability of making four sales, evaluate the term

$${}_8C_4\left(\tfrac{1}{3}\right)^4\left(\tfrac{2}{3}\right)^4$$

in the expansion of $\left(\frac{1}{3} + \frac{2}{3}\right)^8$.

78. To find the probability that the sales representative in Exercise 77 makes four sales if the probability of a sale with any one customer is $\frac{1}{2}$, evaluate the term

$${}_8C_4\left(\tfrac{1}{2}\right)^4\left(\tfrac{1}{2}\right)^4$$

in the expansion of $\left(\frac{1}{2} + \frac{1}{2}\right)^8$.

Model It

79. ***Data Analysis: Water Consumption*** The table shows the per capita consumption of bottled water $f(t)$ (in gallons) in the United States from 1990 through 2003. (Source: Economic Research Service, U.S. Department of Agriculture)

Year	Consumption, $f(t)$
1990	8.0
1991	8.0
1992	9.7
1993	10.3
1994	11.3
1995	12.1
1996	13.0
1997	13.9
1998	15.0
1999	16.4
2000	17.4
2001	18.8
2002	20.7
2003	22.0

(a) Use the *regression* feature of a graphing utility to find a cubic model for the data. Let t represent the year, with $t = 0$ corresponding to 1990.

(b) Use a graphing utility to plot the data and the model in the same viewing window.

(c) You want to adjust the model so that $t = 0$ corresponds to 2000 rather than 1990. To do this, you shift the graph of f 10 units *to the left* to obtain $g(t) = f(t + 10)$. Write $g(t)$ in standard form.

(d) Use a graphing utility to graph g in the same viewing window as f.

(e) Use both models to estimate the per capita consumption of bottled water in 2008. Do you obtain the same answer?

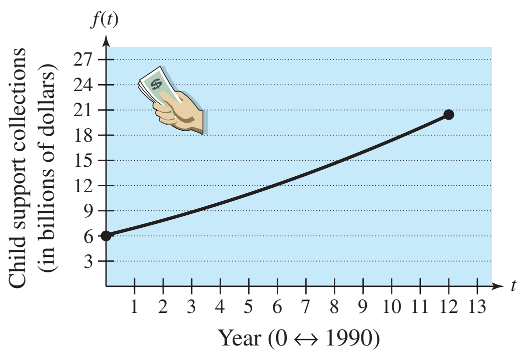
(f) Describe the overall trend in the data. What factors do you think may have contributed to the increase in the per capita consumption of bottled water?

333202_0905.qxd 12/5/05 11:37 AM Page 690

80. ***Child Support*** The amounts $f(t)$ (in billions of dollars) of child support collected in the United States from 1990 to 2002 can be approximated by the model

$$f(t) = 0.031t^2 + 0.82t + 6.1, \quad 0 \le t \le 12$$

where t represents the year, with $t = 0$ corresponding to 1990 (see figure). (Source: U.S. Department of Health and Human Services)



(a) You want to adjust the model so that $t = 0$ corresponds to 2000 rather than 1990. To do this, you shift the graph of f 10 units *to the left* and obtain $g(t) = f(t + 10)$. Write $g(t)$ in standard form.

(b) Use a graphing utility to graph f and g in the same viewing window.

(c) Use the graphs to estimate when the child support collections will exceed \$30 billion.

Synthesis

True or False? **In Exercises 81–83, determine whether the statement is true or false. Justify your answer.**

81. The Binomial Theorem could be used to produce each row of Pascal's Triangle.

82. A binomial that represents a difference cannot always be accurately expanded using the Binomial Theorem.

83. The x^{10}-term and the x^{14}-term of the expansion of $(x^2 + 3)^{12}$ have identical coefficients.

84. ***Writing*** In your own words, explain how to form the rows of Pascal's Triangle.

85. Form rows 8–10 of Pascal's Triangle.

86. ***Think About It*** How many terms are in the expansion of $(x + y)^n$?

87. ***Think About It*** How do the expansions of $(x + y)^n$ and $(x - y)^n$ differ?

88. ***Graphical Reasoning*** Which two functions have identical graphs, and why? Use a graphing utility to graph the functions in the given order and in the same viewing window. Compare the graphs.

(a) $f(x) = (1 - x)^3$

(b) $g(x) = 1 - x^3$

(c) $h(x) = 1 + 3x + 3x^2 + x^3$

(d) $k(x) = 1 - 3x + 3x^2 - x^3$

(e) $p(x) = 1 + 3x - 3x^2 + x^3$

Proof **In Exercises 89–92, prove the property for all integers r and n where $0 \le r \le n$.**

89. ${}_nC_r = {}_nC_{n-r}$

90. ${}_nC_0 - {}_nC_1 + {}_nC_2 - \cdots \pm {}_nC_n = 0$

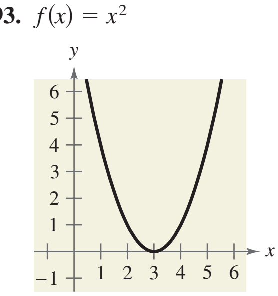
91. ${}_{n+1}C_r = {}_nC_r + {}_nC_{r-1}$

92. The sum of the numbers in the nth row of Pascal's Triangle is 2^n.

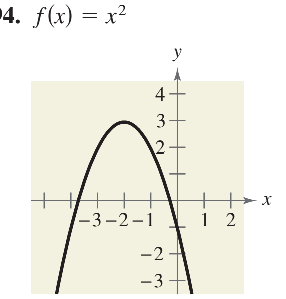
Skills Review

In Exercises 93–96, the graph of $y = g(x)$ is shown. Graph f and use the graph to write an equation for the graph of g.

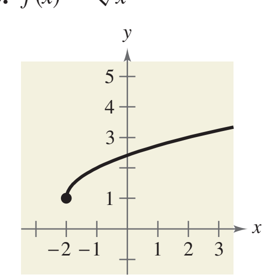
93. $f(x) = x^2$



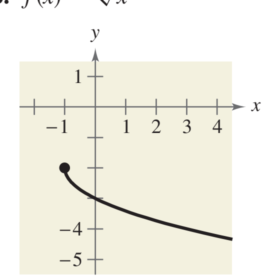
94. $f(x) = x^2$



95. $f(x) = \sqrt{x}$



96. $f(x) = \sqrt{x}$



In Exercises 97 and 98, find the inverse of the matrix.

97. $\begin{bmatrix} -6 & 5 \\ -5 & 4 \end{bmatrix}$

98. $\begin{bmatrix} 1.2 & -2.3 \\ -2 & 4 \end{bmatrix}$