6 Chapter Summary

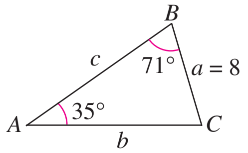
What did you learn?

Section 6.1	Review Exercises
☐ Use the Law of Sines to solve oblique triangles (AAS, ASA, or SSA) *(p. 430, 432)*.	1–12
☐ Find areas of oblique triangles *(p. 434)*.	13–16
☐ Use the Law of Sines to model and solve real-life problems *(p. 435)*.	17–20
Section 6.2	
☐ Use the Law of Cosines to solve oblique triangles (SSS or SAS) *(p. 439)*.	21–28
☐ Use the Law of Cosines to model and solve real-life problems *(p. 441)*.	29–32
☐ Use Heron's Area Formula to find areas of triangles *(p. 442)*.	33–36
Section 6.3	
☐ Represent vectors as directed line segments *(p. 447)*.	37, 38
☐ Write the component forms of vectors *(p. 448)*.	39–44
☐ Perform basic vector operations and represent vectors graphically *(p. 449)*.	45–56
☐ Write vectors as linear combinations of unit vectors *(p. 451)*.	57–62
☐ Find the direction angles of vectors *(p. 453)*.	63–68
☐ Use vectors to model and solve real-life problems *(p. 454)*.	69–72
Section 6.4	
☐ Find the dot product of two vectors and use the properties of the dot product *(p. 460)*.	73–80
☐ Find the angle between two vectors and determine whether two vectors are orthogonal *(p. 461)*.	81–88
☐ Write vectors as sums of two vector components *(p. 463)*.	89–92
☐ Use vectors to find the work done by a force *(p. 466)*.	93–96
Section 6.5	
☐ Plot complex numbers in the complex plane and find absolute values of complex numbers *(p. 470)*.	97–100
☐ Write the trigonometric forms of complex numbers *(p. 471)*.	101–104
☐ Multiply and divide complex numbers written in trigonometric form *(p. 472)*.	105, 106
☐ Use DeMoivre's Theorem to find powers of complex numbers *(p. 474)*	107–110
☐ Find *n*th roots of complex numbers *(p. 475)*.	111–118

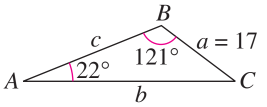
6 Review Exercises

6.1 **In Exercises 1–12, use the Law of Sines to solve (if possible) the triangle. If two solutions exist, find both. Round your answers to two decimal places.**

1.



2.



3. $B = 72°, \ C = 82°, \ b = 54$

4. $B = 10°, \ C = 20°, \ c = 33$

5. $A = 16°, \ B = 98°, \ c = 8.4$

6. $A = 95°, \ B = 45°, \ c = 104.8$

7. $A = 24°, \ C = 48°, \ b = 27.5$

8. $B = 64°, \ C = 36°, \ a = 367$

9. $B = 150°, \ b = 30, \ c = 10$

10. $B = 150°, \ a = 10, \ b = 3$

11. $A = 75°, \ a = 51.2, \ b = 33.7$

12. $B = 25°, \ a = 6.2, \ b = 4$

In Exercises 13–16, find the area of the triangle having the indicated angle and sides.

13. $A = 27°, \ b = 5, \ c = 7$

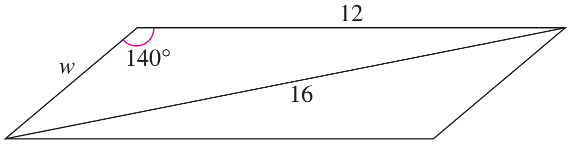
14. $B = 80°, \ a = 4, \ c = 8$

15. $C = 123°, \ a = 16, \ b = 5$

16. $A = 11°, \ b = 22, \ c = 21$

17. ***Height*** From a certain distance, the angle of elevation to the top of a building is 17°. At a point 50 meters closer to the building, the angle of elevation is 31°. Approximate the height of the building.

18. ***Geometry*** Find the length of the side w of the parallelogram.



19. ***Height*** A tree stands on a hillside of slope 28° from the horizontal. From a point 75 feet down the hill, the angle of elevation to the top of the tree is 45° (see figure). Find the height of the tree.

Exercise with no solution: 10   Exercise with two solutions: 12

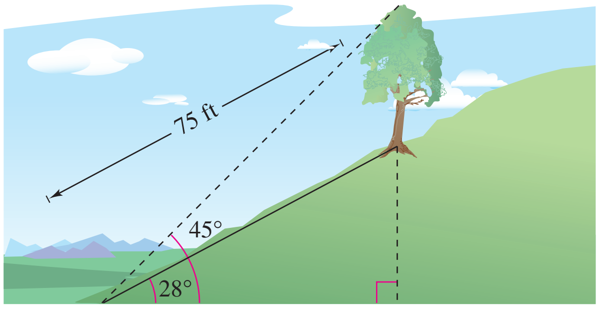


FIGURE FOR 19

20. ***River Width*** A surveyor finds that a tree on the opposite bank of a river, flowing due east, has a bearing of N 22° 30′ E from a certain point and a bearing of N 15° W from a point 400 feet downstream. Find the width of the river.

6.2 **In Exercises 21–28, use the Law of Cosines to solve the triangle. Round your answers to two decimal places.**

21. $a = 5, \ b = 8, \ c = 10$

22. $a = 80, \ b = 60, \ c = 100$

23. $a = 2.5, \ b = 5.0, \ c = 4.5$

24. $a = 16.4, \ b = 8.8, \ c = 12.2$

25. $B = 110°, \ a = 4, \ c = 4$

26. $B = 150°, \ a = 10, \ c = 20$

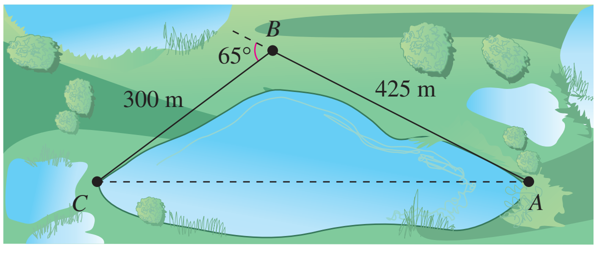
27. $C = 43°, \ a = 22.5, \ b = 31.4$

28. $A = 62°, \ b = 11.34, \ c = 19.52$

29. ***Geometry*** The lengths of the diagonals of a parallelogram are 10 feet and 16 feet. Find the lengths of the sides of the parallelogram if the diagonals intersect at an angle of 28°.

30. ***Geometry*** The lengths of the diagonals of a parallelogram are 30 meters and 40 meters. Find the lengths of the sides of the parallelogram if the diagonals intersect at an angle of 34°.

31. ***Surveying*** To approximate the length of a marsh, a surveyor walks 425 meters from point A to point B. Then the surveyor turns 65° and walks 300 meters to point C (see figure). Approximate the length AC of the marsh.



32. ***Navigation*** Two planes leave Raleigh-Durham Airport at approximately the same time. One is flying 425 miles per hour at a bearing of 355°, and the other is flying 530 miles per hour at a bearing of 67°. Draw a figure that gives a visual representation of the problem and determine the distance between the planes after they have flown for 2 hours.

In Exercises 33–36, use Heron's Area Formula to find the area of the triangle.

33. $a = 4, \ b = 5, \ c = 7$

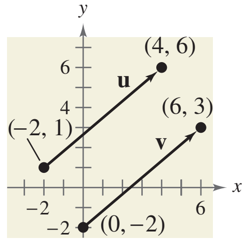
34. $a = 15, \ b = 8, \ c = 10$

35. $a = 12.3, \ b = 15.8, \ c = 3.7$

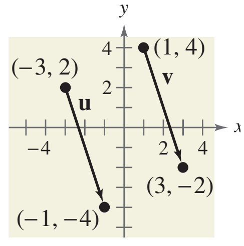
36. $a = 38.1, \ b = 26.7, \ c = 19.4$

6.3 **In Exercises 37 and 38, show that $\mathbf{u} = \mathbf{v}$.**

37.

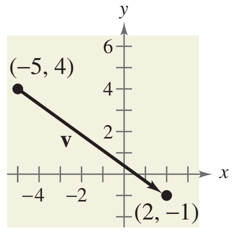


38.

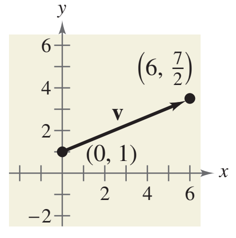


In Exercises 39–44, find the component form of the vector v satisfying the conditions.

39.



40.



41. Initial point: (0, 10); terminal point: (7, 3)

42. Initial point: (1, 5); terminal point: (15, 9)

43. $\|\mathbf{v}\| = 8, \quad \theta = 120°$

44. $\|\mathbf{v}\| = \frac{1}{2}, \quad \theta = 225°$

In Exercises 45–52, find (a) $\mathbf{u} + \mathbf{v}$, (b) $\mathbf{u} - \mathbf{v}$, (c) $3\mathbf{u}$, and (d) $2\mathbf{v} + 5\mathbf{u}$.

45. $\mathbf{u} = \langle -1, -3 \rangle, \mathbf{v} = \langle -3, 6 \rangle$

46. $\mathbf{u} = \langle 4, 5 \rangle, \mathbf{v} = \langle 0, -1 \rangle$

47. $\mathbf{u} = \langle -5, 2 \rangle, \mathbf{v} = \langle 4, 4 \rangle$

48. $\mathbf{u} = \langle 1, -8 \rangle, \mathbf{v} = \langle 3, -2 \rangle$

49. $\mathbf{u} = 2\mathbf{i} - \mathbf{j}, \mathbf{v} = 5\mathbf{i} + 3\mathbf{j}$

50. $\mathbf{u} = -7\mathbf{i} - 3\mathbf{j}, \mathbf{v} = 4\mathbf{i} - \mathbf{j}$

51. $\mathbf{u} = 4\mathbf{i}, \mathbf{v} = -\mathbf{i} + 6\mathbf{j}$

52. $\mathbf{u} = -6\mathbf{j}, \mathbf{v} = \mathbf{i} + \mathbf{j}$

In Exercises 53–56, find the component form of w and sketch the specified vector operations geometrically, where $\mathbf{u} = 6\mathbf{i} - 5\mathbf{j}$ and $\mathbf{v} = 1 - \mathbf{i} + 3\mathbf{j}$.

53. $\mathbf{w} = 2\mathbf{u} + \mathbf{v}$

54. $\mathbf{w} = 4\mathbf{u} - 5\mathbf{v}$

55. $\mathbf{w} = 3\mathbf{v}$

56. $\mathbf{w} = \frac{1}{2}\mathbf{v}$

In Exercises 57–60, write vector u as a linear combination of the standard unit vectors i and j.

57. $\mathbf{u} = \langle -3, 4 \rangle$

58. $\mathbf{u} = \langle -6, -8 \rangle$

59. $\mathbf{u}$ has initial point (3, 4) and terminal point (9, 8).

60. $\mathbf{u}$ has initial point (−2, 7) and terminal point (5, −9).

In Exercises 61 and 62, write the vector v in the form $\|\mathbf{v}\|(\cos\theta\mathbf{i} + \sin\theta\mathbf{j})$.

61. $\mathbf{v} = -10\mathbf{i} + 10\mathbf{j}$

62. $\mathbf{v} = 4\mathbf{i} - \mathbf{j}$

In Exercises 63–68, find the magnitude and the direction angle of the vector v.

63. $\mathbf{v} = 7(\cos 60°\mathbf{i} + \sin 60°\mathbf{j})$

64. $\mathbf{v} = 3(\cos 150°\mathbf{i} + \sin 150°\mathbf{j})$

65. $\mathbf{v} = 5\mathbf{i} + 4\mathbf{j}$

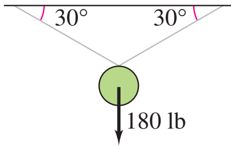
66. $\mathbf{v} = -4\mathbf{i} + 7\mathbf{j}$

67. $\mathbf{v} = -3\mathbf{i} - 3\mathbf{j}$

68. $\mathbf{v} = 8\mathbf{i} - \mathbf{j}$

69. ***Resultant Force*** Forces with magnitudes of 85 pounds and 50 pounds act on a single point. The angle between the forces is 15°. Describe the resultant force.

70. ***Rope Tension*** A 180-pound weight is supported by two ropes, as shown in the figure. Find the tension in each rope.



71. *Navigation* An airplane has an airspeed of 430 miles per hour at a bearing of 135°. The wind velocity is 35 miles per hour in the direction of N 30° E. Find the resultant speed and direction of the airplane.

72. *Navigation* An airplane has an airspeed of 724 kilometers per hour at a bearing of 30°. The wind velocity is 32 kilometers per hour from the west. Find the resultant speed and direction of the airplane.

6.4 In Exercises 73–76, find the dot product of u. and v.

73. $\mathbf{u} = \langle 6, 7\rangle$, $\mathbf{v} = \langle -3, 9\rangle$

74. $\mathbf{u} = \langle -7, 12\rangle$, $\mathbf{v} = \langle -4, -14\rangle$

75. $\mathbf{u} = 3\mathbf{i} + 7\mathbf{j}$, $\mathbf{v} = 11\mathbf{i} - 5\mathbf{j}$

76. $\mathbf{u} = -7\mathbf{i} + 2\mathbf{j}$, $\mathbf{v} = 16\mathbf{i} - 12\mathbf{j}$

In Exercises 77–80, use the vectors $\mathbf{u} = \langle -3, 4\rangle$ and $\mathbf{v} = \langle 2, 1\rangle$ to find the indicated quantity. State whether the result is a vector or a scalar.

77. $2\mathbf{u} \cdot \mathbf{u}$

78. $\|\mathbf{v}\|^2$

79. $\mathbf{u}(\mathbf{u} \cdot \mathbf{v})$

80. $3\mathbf{u} \cdot \mathbf{v}$

In Exercises 81–84, find the angle θ between the vectors.

81. $\mathbf{u} = \cos\dfrac{7\pi}{4}\mathbf{i} + \sin\dfrac{7\pi}{4}\mathbf{j}$

$\mathbf{v} = \cos\dfrac{5\pi}{6}\mathbf{i} + \sin\dfrac{5\pi}{6}\mathbf{j}$

82. $\mathbf{u} = \cos 45°\mathbf{i} + \sin 45°\mathbf{j}$

$\mathbf{v} = \cos 300°\mathbf{i} + \sin 300°\mathbf{j}$

83. $\mathbf{u} = \langle 2\sqrt{2}, -4\rangle, \quad \mathbf{v} = \langle -\sqrt{2}, 1\rangle$

84. $\mathbf{u} = \langle 3, \sqrt{3}\rangle, \quad \mathbf{v} = \langle 4, 3\sqrt{3}\rangle$

In Exercises 85–88, determine whether u and v are orthogonal, parallel, or neither.

85. $\mathbf{u} = \langle -3, 8\rangle$, $\mathbf{v} = \langle 8, 3\rangle$

86. $\mathbf{u} = \langle \frac{1}{4}, -\frac{1}{2}\rangle$, $\mathbf{v} = \langle -2, 4\rangle$

87. $\mathbf{u} = -\mathbf{i}$, $\mathbf{v} = \mathbf{i} + 2\mathbf{j}$

88. $\mathbf{u} = -2\mathbf{i} + \mathbf{j}$, $\mathbf{v} = 3\mathbf{i} + 6\mathbf{j}$

In Exercises 89–92, find the projection of u onto v. Then write u as the sum of two orthogonal vectors, one of which is $\text{proj}_{\mathbf{v}}\mathbf{u}$.

89. $\mathbf{u} = \langle -4, 3\rangle, \ \mathbf{v} = \langle -8, -2\rangle$

90. $\mathbf{u} = \langle 5, 6\rangle, \ \mathbf{v} = \langle 10, 0\rangle$

91. $\mathbf{u} = \langle 2, 7\rangle, \ \mathbf{v} = \langle 1, -1\rangle$

92. $\mathbf{u} = \langle -3, 5\rangle, \ \mathbf{v} = \langle -5, 2\rangle$

Work **In Exercises 93 and 94, find the work done in moving a particle from P to Q if the magnitude and direction of the force are given by v.**

93. $P = (5, 3), Q = (8, 9), \mathbf{v} = \langle 2, 7\rangle$

94. $P = (-2, -9), Q = (-12, 8), \mathbf{v} = 3\mathbf{i} - 6\mathbf{j}$

95. *Work* Determine the work done by a crane lifting an 18,000-pound truck 48 inches.

96. *Work* A mover exerts a horizontal force of 25 pounds on a crate as it is pushed up a ramp that is 12 feet long and inclined at an angle of 20° above the horizontal. Find the work done in pushing the crate.

6.5 In Exercises 97–100, plot the complex number and find its absolute value.

97. $7i$

98. $-6i$

99. $5 + 3i$

100. $-10 - 4i$

In Exercises 101–104, write the complex number in trigonometric form.

101. $5 - 5i$

102. $5 + 12i$

103. $-3\sqrt{3} + 3i$

104. -7

In Exercises 105 and 106, (a) write the two complex numbers in trigonometric form, and (b) use the trigonometric forms to find z_1z_2 and z_1/z_2, where $z_2 \neq 0$.

105. $z_1 = 2\sqrt{3} - 2i, \quad z_2 = -10i$

106. $z_1 = -3(1 + i), \quad z_2 = 2(\sqrt{3} + i)$

In Exercises 107–110, use DeMoivre's Theorem to find the indicated power of the complex number. Write the result in standard form.

107. $\left[5\left(\cos\dfrac{\pi}{12} + i\sin\dfrac{\pi}{12}\right)\right]^4$

108. $\left[2\left(\cos\dfrac{4\pi}{15} + i\sin\dfrac{4\pi}{15}\right)\right]^5$

109. $(2 + 3i)^6$

110. $(1 - i)^8$

In Exercises 111–114, (a) use the theorem on page 476 to find the indicated roots of the complex number, (b) represent each of the roots graphically, and (c) write each of the roots in standard form.

111. Sixth roots of $-729i$

112. Fourth roots of $256i$

113. Cube roots of 8

114. Fifth roots of -1024

In Exercises 115–118, use the theorem on page 476 to find all solutions of the equation and represent the solutions graphically.

115. $x^4 + 81 = 0$

116. $x^5 - 32 = 0$

117. $x^3 + 8i = 0$

118. $(x^3 - 1)(x^2 + 1) = 0$

Synthesis

True or False? **In Exercises 119–123, determine whether the statement is true or false. Justify your answer.**

119. The Law of Sines is true if one of the angles in the triangle is a right angle.

120. When the Law of Sines is used, the solution is always unique.

121. If $\mathbf{u}$ is a unit vector in the direction of $\mathbf{v}$, then $\mathbf{v} = \|\mathbf{v}\|\,\mathbf{u}$.

122. If $\mathbf{v} = a\mathbf{i} + b\mathbf{j} = \mathbf{0}$, then $a = -b$.

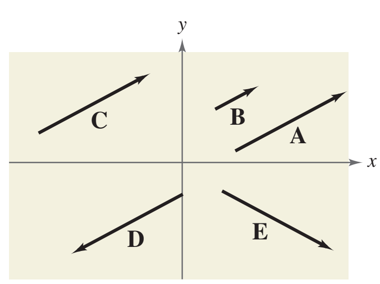
123. $x = \sqrt{3} + i$ is a solution of the equation $x^2 - 8i = 0$.

124. State the Law of Sines from memory.

125. State the Law of Cosines from memory.

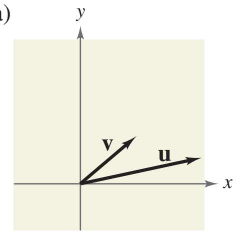
126. What characterizes a vector in the plane?

127. Which vectors in the figure appear to be equivalent?

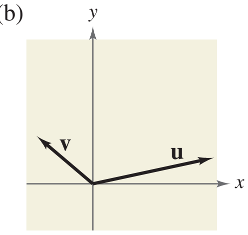


128. The vectors $\mathbf{u}$ and $\mathbf{v}$ have the same magnitudes in the two figures. In which figure will the magnitude of the sum be greater? Give a reason for your answer.

(a)



(b)



129. Give a geometric description of the scalar multiple $k\mathbf{u}$ of the vector $\mathbf{u}$, for $k > 0$ and for $k < 0$.

130. Give a geometric description of the sum of the vectors $\mathbf{u}$ and $\mathbf{v}$.

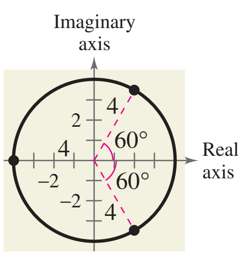
Graphical Reasoning **In Exercises 131 and 132, use the graph of the roots of a complex number.**

(a) Write each of the roots in trigonometric form.

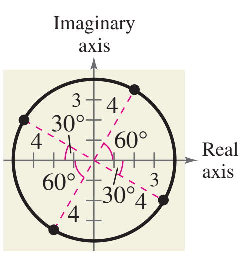
(b) Identify the complex number whose roots are given.

(c) Use a graphing utility to verify the results of part (b).

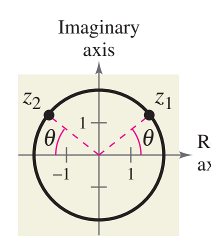
131.



132.



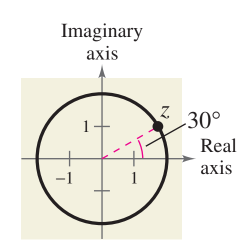
133. The figure shows z_1 and z_2. Describe z_1z_2 and z_1/z_2.



134. One of the fourth roots of a complex number z is shown in the figure.

(a) How many roots are not shown?

(b) Describe the other roots.



6 Chapter Test

Take this test as you would take a test in class. When you are finished, check your work against the answers given in the back of the book.

In Exercises 1–6, use the information to solve the triangle. If two solutions exist, find both solutions. Round your answers to two decimal places.

1. $A = 24°,\ B = 68°,\ a = 12.2$

2. $B = 104°,\ C = 33°,\ a = 18.1$

3. $A = 24°,\ a = 11.2,\ b = 13.4$

4. $a = 4.0,\ b = 7.3,\ c = 12.4$

5. $B = 100°,\ a = 15,\ b = 23$

6. $C = 123°,\ a = 41,\ b = 57$

7. A triangular parcel of land has borders of lengths 60 meters, 70 meters, and 82 meters. Find the area of the parcel of land.

8. An airplane flies 370 miles from point A to point B with a bearing of 24°. It then flies 240 miles from point B to point C with a bearing of 37° (see figure). Find the distance and bearing from point A to point C.

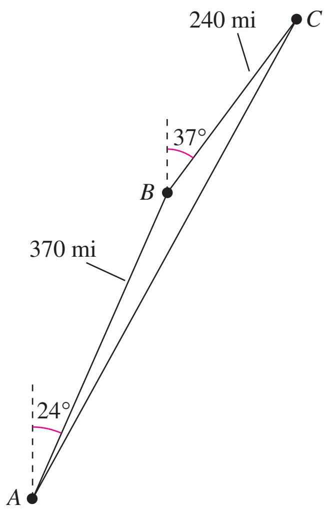


FIGURE FOR 8

In Exercises 9 and 10, find the component form of the vector v satisfying the given conditions.

9. Initial point of $\mathbf{v}$: $(-3, 7)$; terminal point of $\mathbf{v}$: $(11, -16)$

10. Magnitude of $\mathbf{v}$: $\|\mathbf{v}\| = 12$; direction of $\mathbf{v}$: $\mathbf{u} = \langle 3, -5 \rangle$

In Exercises 11–13, $\mathbf{u} = \langle 3, 5 \rangle$ and $\mathbf{v} = \langle -7, 1 \rangle$. Find the resultant vector and sketch its graph.

11. $\mathbf{u} + \mathbf{v}$

12. $\mathbf{u} - \mathbf{v}$

13. $5\mathbf{u} - 3\mathbf{v}$

14. Find a unit vector in the direction of $\mathbf{u} = \langle 4, -3 \rangle$.

15. Forces with magnitudes of 250 pounds and 130 pounds act on an object at angles of 45° and −60°, respectively, with the x-axis. Find the direction and magnitude of the resultant of these forces.

16. Find the angle between the vectors $\mathbf{u} = \langle -1, 5 \rangle$ and $\mathbf{v} = \langle 3, -2 \rangle$.

17. Are the vectors $\mathbf{u} = \langle 6, 10 \rangle$ and $\mathbf{v} = \langle 2, 3 \rangle$ orthogonal?

18. Find the projection of $\mathbf{u} = \langle 6, 7 \rangle$ onto $\mathbf{v} = \langle -5, -1 \rangle$. Then write $\mathbf{u}$ as the sum of two orthogonal vectors.

19. A 500-pound motorcycle is headed up a hill inclined at 12°. What force is required to keep the motorcycle from rolling down the hill when stopped at a red light?

20. Write the complex number $z = 5 - 5i$ in trigonometric form.

21. Write the complex number $z = 6(\cos 120° + i \sin 120°)$ in standard form.

In Exercises 22 and 23, use DeMoivre's Theorem to find the indicated power of the complex number. Write the result in standard form.

22. $\left[3\left(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6}\right)\right]^8$

23. $(3 - 3i)^6$

24. Find the fourth roots of $256\left(1 + \sqrt{3}\,i\right)$.

25. Find all solutions of the equation $x^3 - 27i = 0$ and represent the solutions graphically.

6 Cumulative Test for Chapter 4–6

Take this test to review the material from earlier chapters. When you are finished, check your work against the answers given in the back of the book.

1. Consider the angle $\theta = -120°$.

(a) Sketch the angle in standard position.

(b) Determine a coterminal angle in the interval $[0°, 360°)$.

(c) Convert the angle to radian measure.

(d) Find the reference angle θ'.

(e) Find the exact values of the six trigonometric functions of θ.

2. Convert the angle $\theta = 2.35$ radians to degrees. Round the answer to one decimal place.

3. Find $\cos\theta$ if $\tan\theta = -\frac{4}{3}$ and $\sin\theta < 0$.

In Exercises 4–6, sketch the graph of the function. (Include two full periods.)

4. $f(x) = 3 - 2\sin\pi x$ **5.** $g(x) = \dfrac{1}{2}\tan\left(x - \dfrac{\pi}{2}\right)$ **6.** $h(x) = -\sec(x + \pi)$

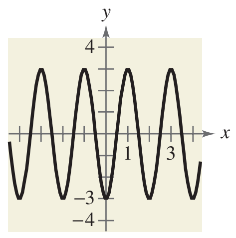


FIGURE FOR 7

7. Find a, b, and c such that the graph of the function $h(x) = a\cos(bx + c)$ matches the graph in the figure.

8. Sketch the graph of the function $f(x) = \frac{1}{2}x\sin x$ over the interval $-3\pi \le x \le 3\pi$.

In Exercises 9 and 10, find the exact value of the expression without using a calculator.

9. $\tan(\arctan 6.7)$ **10.** $\tan\left(\arcsin\frac{3}{5}\right)$

11. Write an algebraic expression equivalent to $\sin(\arccos 2x)$.

12. Use the fundamental identities to simplify: $\cos\left(\dfrac{\pi}{2} - x\right)\csc x$.

13. Subtract and simplify: $\dfrac{\sin\theta - 1}{\cos\theta} - \dfrac{\cos\theta}{\sin\theta - 1}$.

In Exercises 14–16, verify the identity.

14. $\cot^2\alpha(\sec^2\alpha - 1) = 1$

15. $\sin(x + y)\sin(x - y) = \sin^2 x - \sin^2 y$

16. $\sin^2 x\cos^2 x = \frac{1}{8}(1 - \cos 4x)$

In Exercises 17 and 18, find all solutions of the equation in the interval $[0, 2\pi)$.

17. $2\cos^2\beta - \cos\beta = 0$

18. $3\tan\theta - \cot\theta = 0$

19. Use the Quadratic Formula to solve the equation in the interval $[0, 2\pi)$: $\sin^2 x + 2\sin x + 1 = 0$.

20. Given that $\sin u = \frac{12}{13}$, $\cos v = \frac{3}{5}$, and angles u and v are both in Quadrant I, find $\tan(u - v)$.

21. If $\tan\theta = \frac{1}{2}$, find the exact value of $\tan(2\theta)$.

22. If $\tan \theta = \frac{4}{3}$, find the exact value of $\sin \frac{\theta}{2}$.

23. Write the product $5 \sin \frac{3\pi}{4} \cdot \cos \frac{7\pi}{4}$ as a sum or difference.

24. Write $\cos 8x + \cos 4x$ as a product.

In Exercises 25–28, use the information to solve the triangle shown in the figure. Round your answers to two decimal places.

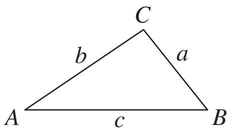


FIGURE FOR 25–28

25. $A = 30°, \ a = 9, \ b = 8$

26. $A = 30°, b = 8, \ c = 10$

27. $A = 30°, \ C = 90°, \ b = 10$

28. $a = 4, \ b = 8, \ c = 9$

29. Two sides of a triangle have lengths 7 inches and 12 inches. Their included angle measures 60°. Find the area of the triangle.

30. Find the area of a triangle with sides of lengths 11 inches, 16 inches, and 17 inches.

31. Write the vector $\mathbf{u} = \langle 3, 5 \rangle$ as a linear combination of the standard unit vectors $\mathbf{i}$ and $\mathbf{j}$.

32. Find a unit vector in the direction of $\mathbf{v} = \mathbf{i} + \mathbf{j}$.

33. Find $\mathbf{u} \cdot \mathbf{v}$ for $\mathbf{u} = 3\mathbf{i} + 4\mathbf{j}$ and $\mathbf{v} = \mathbf{i} - 2\mathbf{j}$.

34. Find the projection of $\mathbf{u} = \langle 8, -2 \rangle$ onto $\mathbf{v} = \langle 1, 5 \rangle$. Then write $\mathbf{u}$ as the sum of two orthogonal vectors.

35. Write the complex number $-2 \ + 2i$ in trigonometric form.

36. Find the product of $[4(\cos 30° + i \sin 30°)][6(\cos 120° + i \sin 120°)]$. Write the answer in standard form.

37. Find the three cube roots of 1.

38. Find all the solutions of the equation $x^5 + 243 = 0$.

39. A ceiling fan with 21-inch blades makes 63 revolutions per minute. Find the angular speed of the fan in radians per minute. Find the linear speed of the tips of the blades in inches per minute.

40. Find the area of the sector of a circle with a radius of 8 yards and a central angle of 114°.

41. From a point 200 feet from a flagpole, the angles of elevation to the bottom and top of the flag are $16° \, 45'$ and 18°, respectively. Approximate the height of the flag to the nearest foot.

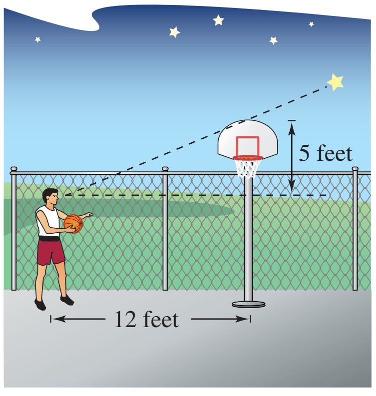


FIGURE FOR 42

42. To determine the angle of elevation of a star in the sky, you get the star in your line of vision with the backboard of a basketball hoop that is 5 feet higher than your eyes (see figure). Your horizontal distance from the backboard is 12 feet. What is the angle of elevation of the star?

43. Write a model for a particle in simple harmonic motion with a displacement of 4 inches and a period of 8 seconds.

44. An airplane's velocity with respect to the air is 500 kilometers per hour, with a bearing of 30°. The wind at the altitude of the plane has a velocity of 50 kilometers per hour with a bearing of N 60° E. What is the true direction of the plane, and what is its speed relative to the ground?

45. A force of 85 pounds exerted at an angle of 60° above the horizontal is required to slide an object across a floor. The object is dragged 10 feet. Determine the work done in sliding the object.

Proofs in Mathematics

Law of Tangents

Besides the Law of Sines and the Law of Cosines, there is also a Law of Tangents, which was developed by Francois Vìete (1540–1603). The Law of Tangents follows from the Law of Sines and the sum-to-product formulas for sine and is defined as follows.

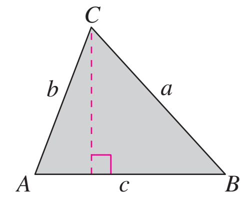
$$\frac{a+b}{a-b} = \frac{\tan[(A+B)/2]}{\tan[(A-B)/2]}$$

The Law of Tangents can be used to solve a triangle when two sides and the included angle are given (SAS). Before calculators were invented, the Law of Tangents was used to solve the SAS case instead of the Law of Cosines, because computation with a table of tangent values was easier.

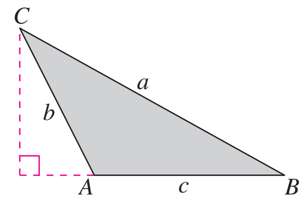
Law of Sines *(p. 430)*

If ABC is a triangle with sides a, b, and c, then

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$



A is acute.



A is obtuse.

Proof

Let h be the altitude of either triangle found in the figure above. Then you have

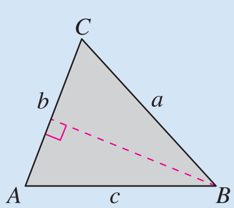
$$\sin A = \frac{h}{b} \quad \text{or} \quad h = b \sin A$$

$$\sin B = \frac{h}{a} \quad \text{or} \quad h = a \sin B.$$

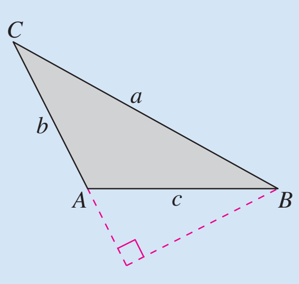
Equating these two values of h, you have

$$a \sin B = b \sin A \quad \text{or} \quad \frac{a}{\sin A} = \frac{b}{\sin B}.$$

Note that $\sin A \neq 0$ and $\sin B \neq 0$ because no angle of a triangle can have a measure of $0°$ or $180°$. In a similar manner, construct an altitude from vertex B to side AC (extended in the obtuse triangle), as shown at the left. Then you have



A is acute.



A is obtuse.

$$\sin A = \frac{h}{c} \quad \text{or} \quad h = c \sin A$$

$$\sin C = \frac{h}{a} \quad \text{or} \quad h = a \sin C.$$

Equating these two values of h, you have

$$a \sin C = c \sin A \quad \text{or} \quad \frac{a}{\sin A} = \frac{c}{\sin C}.$$

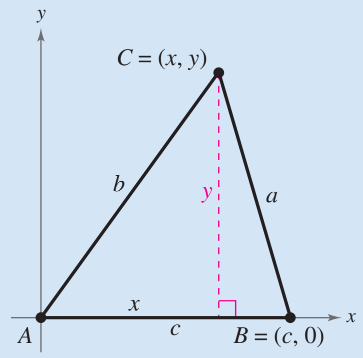
By the Transitive Property of Equality you know that

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

So, the Law of Sines is established.

Law of Cosines *(p. 439)*

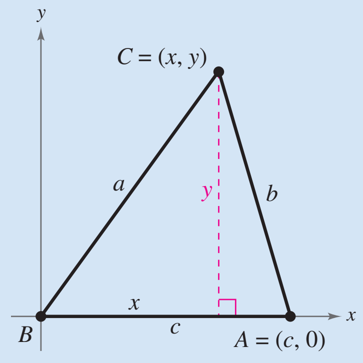
Standard Form	*Alternative Form*
$a^2 = b^2 + c^2 - 2bc \cos A$	$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$
$b^2 = a^2 + c^2 - 2ac \cos B$	$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$
$c^2 = a^2 + b^2 - 2ab \cos C$	$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$



Proof

To prove the first formula, consider the top triangle at the left, which has three acute angles. Note that vertex B has coordinates $(c, 0)$. Furthermore, C has coordinates (x, y), where $x = b \cos A$ and $y = b \sin A$. Because a is the distance from vertex C to vertex B, it follows that

$$
\begin{aligned}
a &= \sqrt{(x - c)^2 + (y - 0)^2} && \text{Distance Formula} \\
a^2 &= (x - c)^2 + (y - 0)^2 && \text{Square each side.} \\
a^2 &= (b \cos A - c)^2 + (b \sin A)^2 && \text{Substitute for } x \text{ and } y. \\
a^2 &= b^2 \cos^2 A - 2bc \cos A + c^2 + b^2 \sin^2 A && \text{Expand.} \\
a^2 &= b^2(\sin^2 A + \cos^2 A) + c^2 - 2bc \cos A && \text{Factor out } b^2. \\
a^2 &= b^2 + c^2 - 2bc \cos A. && \sin^2 A + \cos^2 A = 1
\end{aligned}
$$



To prove the second formula, consider the bottom triangle at the left, which also has three acute angles. Note that vertex A has coordinates $(c, 0)$. Furthermore, C has coordinates (x, y), where $x = a \cos B$ and $y = a \sin B$. Because b is the distance from vertex C to vertex A, it follows that

$$
\begin{aligned}
b &= \sqrt{(x - c)^2 + (y - 0)^2} && \text{Distance Formula} \\
b^2 &= (x - c)^2 + (y - 0)^2 && \text{Square each side.} \\
b^2 &= (a \cos B - c)^2 + (a \sin B)^2 && \text{Substitute for } x \text{ and } y. \\
b^2 &= a^2 \cos^2 B - 2ac \cos B + c^2 + a^2 \sin^2 B && \text{Expand.} \\
b^2 &= a^2(\sin^2 B + \cos^2 B) + c^2 - 2ac \cos B && \text{Factor out } a^2. \\
b^2 &= a^2 + c^2 - 2ac \cos B. && \sin^2 B + \cos^2 B = 1
\end{aligned}
$$

A similar argument is used to establish the third formula.

Heron's Area Formula (p. 442)

Given any triangle with sides of lengths a, b, and c, the area of the triangle is

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \dfrac{(a+b+c)}{2}$.

Proof

From Section 6.1, you know that

$$\text{Area} = \frac{1}{2}bc \sin A \qquad \text{Formula for the area of an oblique triangle}$$

$$(\text{Area})^2 = \frac{1}{4}b^2c^2 \sin^2 A \qquad \text{Square each side.}$$

$$\text{Area} = \sqrt{\frac{1}{4}b^2c^2 \sin^2 A} \qquad \text{Take the square root of each side.}$$

$$= \sqrt{\frac{1}{4}b^2c^2(1-\cos^2 A)} \qquad \text{Pythagorean Identity}$$

$$= \sqrt{\left[\frac{1}{2}bc(1+\cos A)\right]\left[\frac{1}{2}bc(1-\cos A)\right]}. \qquad \text{Factor.}$$

Using the Law of Cosines, you can show that

$$\frac{1}{2}bc(1+\cos A) = \frac{a+b+c}{2} \cdot \frac{-a+b+c}{2}$$

and

$$\frac{1}{2}bc(1-\cos A) = \frac{a-b+c}{2} \cdot \frac{a+b-c}{2}.$$

Letting $s = (a+b+c)/2$, these two equations can be rewritten as

$$\frac{1}{2}bc(1+\cos A) = s(s-a)$$

and

$$\frac{1}{2}bc(1-\cos A) = (s-b)(s-c).$$

By substituting into the last formula for area, you can conclude that

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}.$$

Properties of the Dot Product (p. 460)

Let **u**, **v**, and **w** be vectors in the plane or in space and let c be a scalar.

1. $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$
2. $\mathbf{0} \cdot \mathbf{v} = 0$
3. $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$
4. $\mathbf{v} \cdot \mathbf{v} = \|\mathbf{v}\|^2$
5. $c(\mathbf{u} \cdot \mathbf{v}) = c\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot c\mathbf{v}$

Proof

Let $\mathbf{u} = \langle u_1, u_2 \rangle$, $\mathbf{v} = \langle v_1, v_2 \rangle$, $\mathbf{w} = \langle w_1, w_2 \rangle$, $\mathbf{0} = \langle 0, 0 \rangle$, and let c be a scalar.

1. $\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2 = v_1u_1 + v_2u_2 = \mathbf{v} \cdot \mathbf{u}$

2. $\mathbf{0} \cdot \mathbf{v} = 0 \cdot v_1 + 0 \cdot v_2 = 0$

3.
$$\begin{aligned}\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) &= \mathbf{u} \cdot \langle v_1 + w_1, v_2 + w_2 \rangle\\ &= u_1(v_1 + w_1) + u_2(v_2 + w_2)\\ &= u_1v_1 + u_1w_1 + u_2v_2 + u_2w_2\\ &= (u_1v_1 + u_2v_2) + (u_1w_1 + u_2w_2) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}\end{aligned}$$

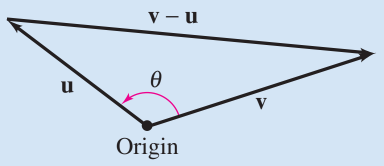
4. $\mathbf{v} \cdot \mathbf{v} = v_1^2 + v_2^2 = \left(\sqrt{v_1^2 + v_2^2}\right)^2 = \|\mathbf{v}\|^2$

5.
$$\begin{aligned}c(\mathbf{u} \cdot \mathbf{v}) &= c(\langle u_1, u_2 \rangle \cdot \langle v_1, v_2 \rangle)\\ &= c(u_1v_1 + u_2v_2)\\ &= (cu_1)v_1 + (cu_2)v_2\\ &= \langle cu_1, cu_2 \rangle \cdot \langle v_1, v_2 \rangle\\ &= c\mathbf{u} \cdot \mathbf{v}\end{aligned}$$

Angle Between Two Vectors (p. 461)

If θ is the angle between two nonzero vectors **u** and **v**, then $\cos\theta = \dfrac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|}$.

Proof



Consider the triangle determined by vectors **u, v,** and **v** − **u**, as shown in the figure. By the Law of Cosines, you can write

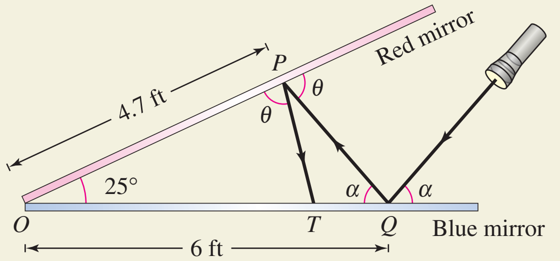
$$\begin{aligned}\|\mathbf{v} - \mathbf{u}\|^2 &= \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 - 2\|\mathbf{u}\|\,\|\mathbf{v}\|\cos\theta\\ (\mathbf{v} - \mathbf{u}) \cdot (\mathbf{v} - \mathbf{u}) &= \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 - 2\|\mathbf{u}\|\,\|\mathbf{v}\|\cos\theta\\ (\mathbf{v} - \mathbf{u}) \cdot \mathbf{v} - (\mathbf{v} - \mathbf{u}) \cdot \mathbf{u} &= \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 - 2\|\mathbf{u}\|\,\|\mathbf{v}\|\cos\theta\\ \mathbf{v} \cdot \mathbf{v} - \mathbf{u} \cdot \mathbf{v} - \mathbf{v} \cdot \mathbf{u} + \mathbf{u} \cdot \mathbf{u} &= \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 - 2\|\mathbf{u}\|\,\|\mathbf{v}\|\cos\theta\\ \|\mathbf{v}\|^2 - 2\mathbf{u} \cdot \mathbf{v} + \|\mathbf{u}\|^2 &= \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 - 2\|\mathbf{u}\|\,\|\mathbf{v}\|\cos\theta\end{aligned}$$

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|}.$$

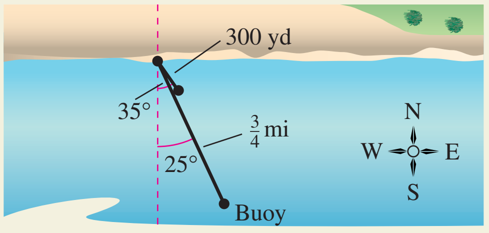
P.S. Problem Solving

This collection of thought-provoking and challenging exercises further explores and expands upon concepts learned in this chapter.

1. In the figure, a beam of light is directed at the blue mirror, reflected to the red mirror, and then reflected back to the blue mirror. Find the distance PT that the light travels from the red mirror back to the blue mirror.



2. A triathlete sets a course to swim S 25° E from a point on shore to a buoy $\frac{3}{4}$ mile away. After swimming 300 yards through a strong current, the triathlete is off course at a bearing of S 35° E. Find the bearing and distance the triathlete needs to swim to correct her course.



3. A hiking party is lost in a national park. Two ranger stations have received an emergency SOS signal from the party. Station B is 75 miles due east of station A. The bearing from station A to the signal is S 60° E and the bearing from station B to the signal is S 75° W.

(a) Draw a diagram that gives a visual representation of the problem.

(b) Find the distance from each station to the SOS signal.

(c) A rescue party is in the park 20 miles from station A at a bearing of S 80° E. Find the distance and the bearing the rescue party must travel to reach the lost hiking party.

4. You are seeding a triangular courtyard. One side of the courtyard is 52 feet long and another side is 46 feet long. The angle opposite the 52-foot side is 65°.

(a) Draw a diagram that gives a visual representation of the problem.

(b) How long is the third side of the courtyard?

(c) One bag of grass covers an area of 50 square feet. How many bags of grass will you need to cover the courtyard?

5. For each pair of vectors, find the following.

(i) $\|\mathbf{u}\|$ (ii) $\|\mathbf{v}\|$ (iii) $\|\mathbf{u} + \mathbf{v}\|$

(iv) $\left\|\dfrac{\mathbf{u}}{\|\mathbf{u}\|}\right\|$ (v) $\left\|\dfrac{\mathbf{v}}{\|\mathbf{v}\|}\right\|$ (vi) $\left\|\dfrac{\mathbf{u} + \mathbf{v}}{\|\mathbf{u} + \mathbf{v}\|}\right\|$

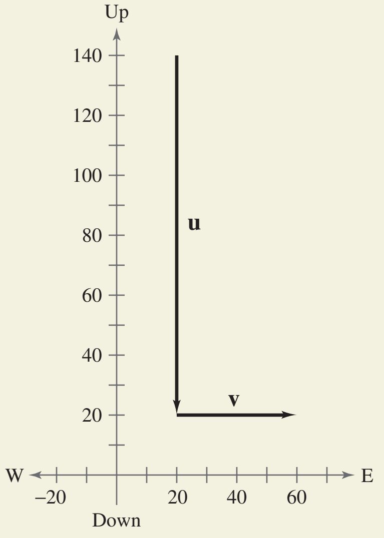
(a) $\mathbf{u} = \langle 1, -1 \rangle$
$\mathbf{v} = \langle -1, 2 \rangle$

(b) $\mathbf{u} = \langle 0, 1 \rangle$
$\mathbf{v} = \langle 3, -3 \rangle$

(c) $\mathbf{u} = \left\langle 1, \frac{1}{2} \right\rangle$
$\mathbf{v} = \langle 2, 3 \rangle$

(d) $\mathbf{u} = \langle 2, -4 \rangle$
$\mathbf{v} = \langle 5, 5 \rangle$

6. A skydiver is falling at a constant downward velocity of 120 miles per hour. In the figure, vector $\mathbf{u}$ represents the skydiver's velocity. A steady breeze pushes the skydiver to the east at 40 miles per hour. Vector $\mathbf{v}$ represents the wind velocity.



(a) Write the vectors $\mathbf{u}$ and $\mathbf{v}$ in component form.

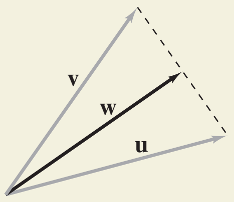
(b) Let $\mathbf{s} = \mathbf{u} + \mathbf{v}$. Use the figure to sketch $\mathbf{s}$. To print an enlarged copy of the graph, go to the website, *www.mathgraphs.com*.

(c) Find the magnitude of $\mathbf{s}$. What information does the magnitude give you about the skydiver's fall?

(d) If there were no wind, the skydiver would fall in a path perpendicular to the ground. At what angle to the ground is the path of the skydiver when the skydiver is affected by the 40 mile per hour wind from due west?

(e) The skydiver is blown to the west at 30 miles per hour. Draw a new figure that gives a visual representation of the problem and find the skydiver's new velocity.

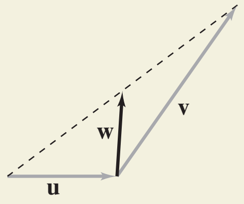
7. Write the vector **w** in terms of **u** and **v**, given that the terminal point of **w** bisects the line segment (see figure).



8. Prove that if **u** is orthogonal to **v** and **w**, then **u** is orthogonal to

$c\mathbf{v} + d\mathbf{w}$

for any scalars c and d (see figure).



9. Two forces of the same magnitude $\mathbf{F}_1$ and $\mathbf{F}_2$ act at angles θ_1 and θ_2, respectively. Use a diagram to compare the work done by $\mathbf{F}_1$ with the work done by $\mathbf{F}_2$ in moving along the vector PQ if

(a) $\theta_1 = -\theta_2$

(b) $\theta_1 = 60°$ and $\theta_2 = 30°$.

10. Four basic forces are in action during flight: weight, lift, thrust, and drag. To fly through the air, an object must overcome its own *weight*. To do this, it must create an upward force called *lift*. To generate lift, a forward motion called *thrust* is needed. The thrust must be great enough to overcome air resistance, which is called *drag*.

For a commercial jet aircraft, a quick climb is important to maximize efficiency, because the performance of an aircraft at high altitudes is enhanced. In addition, it is necessary to clear obstacles such as buildings and mountains and reduce noise in residential areas. In the diagram, the angle θ is called the climb angle. The velocity of the plane can be represented by a vector **v** with a vertical component $\|\mathbf{v}\| \sin \theta$ (called climb speed) and a horizontal component $\|\mathbf{v}\| \cos \theta$, where $\|\mathbf{v}\|$ is the speed of the plane. When taking off, a pilot must decide how much of the thrust to apply to each component. The more the thrust is applied to the horizontal component, the faster the airplane will gain speed. The more the thrust is applied to the vertical component, the quicker the airplane will climb.

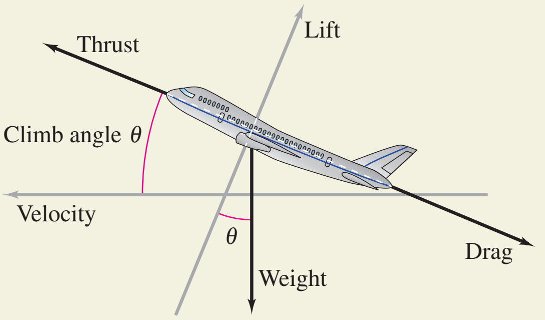


FIGURE FOR 10

(a) Complete the table for an airplane that has a speed of $\|\mathbf{v}\| = 100$ miles per hour.

θ	0.5°	1.0°	1.5°	2.0°	2.5°	3.0°
$\|\mathbf{v}\| \sin \theta$						
$\|\mathbf{v}\| \cos \theta$						

(b) Does an airplane's speed equal the sum of the vertical and horizontal components of its velocity? If not, how could you find the speed of an airplane whose velocity components were known?

(c) Use the result of part (b) to find the speed of an airplane with the given velocity components.

(i) $\|\mathbf{v}\| \sin \theta = 5.235$ miles per hour
$\|\mathbf{v}\| \cos \theta = 149.909$ miles per hour

(ii) $\|\mathbf{v}\| \sin \theta = 10.463$ miles per hour
$\|\mathbf{v}\| \cos \theta = 149.634$ miles per hour