

## 4 Chapter Summary

### What did you learn?

#### Section 4.1

- ☐ Describe angles (p. 282).
- ☐ Use radian measure (p. 283).
- ☐ Use degree measure (p. 285).
- ☐ Use angles to model and solve real-life problems (p. 287).

#### Section 4.2

- ☐ Identify a unit circle and describe its relationship to real numbers (p. 294).
- ☐ Evaluate trigonometric functions using the unit circle (p. 295).
- ☐ Use domain and period to evaluate sine and cosine functions (p. 297).
- ☐ Use a calculator to evaluate trigonometric functions (p. 298).

#### Section 4.3

- ☐ Evaluate trigonometric functions of acute angles (p. 301).
- ☐ Use the fundamental trigonometric identities (p. 304).
- ☐ Use a calculator to evaluate trigonometric functions (p. 305).
- ☐ Use trigonometric functions to model and solve real-life problems (p. 306).

#### Section 4.4

- ☐ Evaluate trigonometric functions of any angle (p. 312).
- ☐ Use reference angles to evaluate trigonometric functions (p. 314).
- ☐ Evaluate trigonometric functions of real numbers (p. 315).

#### Section 4.5

- ☐ Use amplitude and period to help sketch the graphs of sine and cosine functions (p. 323).
- ☐ Sketch translations of the graphs of sine and cosine functions (p. 325).
- ☐ Use sine and cosine functions to model real-life data (p. 327).

#### Section 4.6

- ☐ Sketch the graphs of tangent (p. 332) and cotangent (p. 334) functions.
- ☐ Sketch the graphs of secant and cosecant functions (p. 335).
- ☐ Sketch the graphs of damped trigonometric functions (p. 337).

#### Section 4.7

- ☐ Evaluate and graph the inverse sine function (p. 343).
- ☐ Evaluate and graph the other inverse trigonometric functions (p. 345).
- ☐ Evaluate compositions of trigonometric functions (p. 347).

#### Section 4.8

- ☐ Solve real-life problems involving right triangles (p. 353).
- ☐ Solve real-life problems involving directional bearings (p. 355).
- ☐ Solve real-life problems involving harmonic motion (p. 356).

#### Review Exercises

- 1, 2
- 3–6, 11–18
- 7–18
- 19–24

- 25–28
- 29–32
- 33–36
- 37–40

- 41–44
- 45–48
- 49–54
- 55, 56

- 57–70
- 71–82
- 83–88

- 89–92
- 93–96
- 97, 98

- 99–102
- 103–106
- 107, 108

- 109–114, 123, 126
- 115–122, 124, 125
- 127–132

- 133, 134
- 135
- 136

## 4 Review Exercises

**4.1** In Exercises 1 and 2, estimate the angle to the nearest one-half radian.



In Exercises 3–10, (a) sketch the angle in standard position, (b) determine the quadrant in which the angle lies, and (c) determine one positive and one negative coterminal angle.

- |                      |                       |
|----------------------|-----------------------|
| 3. $\frac{11\pi}{4}$ | 4. $\frac{2\pi}{9}$   |
| 5. $-\frac{4\pi}{3}$ | 6. $-\frac{23\pi}{3}$ |
| 7. $70^\circ$        | 8. $280^\circ$        |
| 9. $-110^\circ$      | 10. $-405^\circ$      |

In Exercises 11–14, convert the angle measure from degrees to radians. Round your answer to three decimal places.

- |                     |                     |
|---------------------|---------------------|
| 11. $480^\circ$     | 12. $-127.5^\circ$  |
| 13. $-33^\circ 45'$ | 14. $196^\circ 77'$ |

In Exercises 15–18, convert the angle measure from radians to degrees. Round your answer to three decimal places.

- |                      |                        |
|----------------------|------------------------|
| 15. $\frac{5\pi}{7}$ | 16. $-\frac{11\pi}{6}$ |
| 17. $-3.5$           | 18. $5.7$              |

**19. Arc Length** Find the length of the arc on a circle with a radius of 20 inches intercepted by a central angle of  $138^\circ$ .

**20. Arc Length** Find the length of the arc on a circle with a radius of 11 meters intercepted by a central angle of  $60^\circ$ .

**21. Phonograph** Compact discs have all but replaced phonograph records. Phonograph records are vinyl discs that rotate on a turntable. A typical record album is 12 inches in diameter and plays at  $33\frac{1}{3}$  revolutions per minute.

- What is the angular speed of a record album?
- What is the linear speed of the outer edge of a record album?

**22. Bicycle** At what speed is a bicyclist traveling when his 27-inch-diameter tires are rotating at an angular speed of  $5\pi$  radians per second?

**23. Circular Sector** Find the area of the sector of a circle with a radius of 18 inches and central angle  $\theta = 120^\circ$ .

**24. Circular Sector** Find the area of the sector of a circle with a radius of 6.5 millimeters and central angle  $\theta = 5\pi/6$ .

**4.2** In Exercises 25–28, find the point  $(x, y)$  on the unit circle that corresponds to the real number  $t$ .

- |                          |                           |
|--------------------------|---------------------------|
| 25. $t = \frac{2\pi}{3}$ | 26. $t = \frac{3\pi}{4}$  |
| 27. $t = \frac{5\pi}{6}$ | 28. $t = -\frac{4\pi}{3}$ |

In Exercises 29–32, evaluate (if possible) the six trigonometric functions of the real number.

- |                           |                         |
|---------------------------|-------------------------|
| 29. $t = \frac{7\pi}{6}$  | 30. $t = \frac{\pi}{4}$ |
| 31. $t = -\frac{2\pi}{3}$ | 32. $t = 2\pi$          |

In Exercises 33–36, evaluate the trigonometric function using its period as an aid.

- |   |   |
|---|---|
| 33. $\sin \frac{11\pi}{4}$              | 34. $\cos 4\pi$                         |
| 35. $\sin\left(-\frac{17\pi}{6}\right)$ | 36. $\cos\left(-\frac{13\pi}{3}\right)$ |

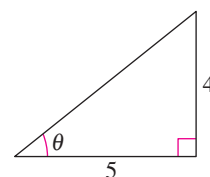


In Exercises 37–40, use a calculator to evaluate the trigonometric function. Round your answer to four decimal places.

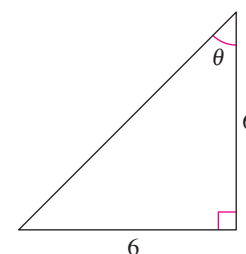
- |                            |                                       |
|----------------------------|---------------------------------------|
| 37. $\tan 33$              | 38. $\csc 10.5$                       |
| 39. $\sec \frac{12\pi}{5}$ | 40. $\sin\left(-\frac{\pi}{9}\right)$ |

**4.3** In Exercises 41–44, find the exact values of the six trigonometric functions of the angle  $\theta$  shown in the figure.

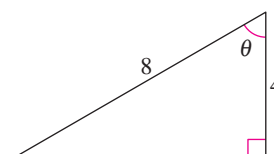
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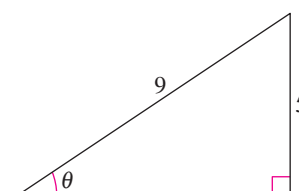
42.



43.



44.



## 366 Chapter 4 Trigonometry

In Exercises 45–48, use the given function value and trigonometric identities (including the cofunction identities) to find the indicated trigonometric functions.

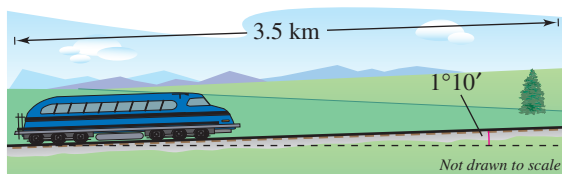
- |                                 |                   |                               |
|---------------------------------|-------------------|-------------------------------|
| 45. $\sin \theta = \frac{1}{3}$ | (a) $\csc \theta$ | (b) $\cos \theta$             |
|                                 | (c) $\sec \theta$ | (d) $\tan \theta$             |
| 46. $\tan \theta = 4$           | (a) $\cot \theta$ | (b) $\sec \theta$             |
|                                 | (c) $\cos \theta$ | (d) $\csc \theta$             |
| 47. $\csc \theta = 4$           | (a) $\sin \theta$ | (b) $\cos \theta$             |
|                                 | (c) $\sec \theta$ | (d) $\tan \theta$             |
| 48. $\csc \theta = 5$           | (a) $\sin \theta$ | (b) $\cot \theta$             |
|                                 | (c) $\tan \theta$ | (d) $\sec(90^\circ - \theta)$ |



In Exercises 49–54, use a calculator to evaluate the trigonometric function. Round your answer to four decimal places.

49.  $\tan 33^\circ$   
 50.  $\csc 11^\circ$   
 51.  $\sin 34.2^\circ$   
 52.  $\sec 79.3^\circ$   
 53.  $\cot 15^\circ 14'$   
 54.  $\cos 78^\circ 11' 58''$

55. **Railroad Grade** A train travels 3.5 kilometers on a straight track with a grade of  $1^\circ 10'$  (see figure). What is the vertical rise of the train in that distance?



56. **Guy Wire** A guy wire runs from the ground to the top of a 25-foot telephone pole. The angle formed between the wire and the ground is  $52^\circ$ . How far from the base of the pole is the wire attached to the ground?

**4.4** In Exercises 57–64, the point is on the terminal side of an angle  $\theta$  in standard position. Determine the exact values of the six trigonometric functions of the angle  $\theta$ .

57.  $(12, 16)$   
 58.  $(3, -4)$   
 59.  $(\frac{2}{3}, \frac{5}{2})$   
 60.  $(-\frac{10}{3}, -\frac{2}{3})$   
 61.  $(-0.5, 4.5)$   
 62.  $(0.3, 0.4)$   
 63.  $(x, 4x), x > 0$   
 64.  $(-2x, -3x), x > 0$

In Exercises 65–70, find the values of the six trigonometric functions of  $\theta$ .

Function Value	Constraint
65. $\sec \theta = \frac{6}{5}$	$\tan \theta < 0$
66. $\csc \theta = \frac{3}{2}$	$\cos \theta < 0$
67. $\sin \theta = \frac{3}{8}$	$\cos \theta < 0$
68. $\tan \theta = \frac{5}{4}$	$\cos \theta < 0$
69. $\cos \theta = -\frac{2}{5}$	$\sin \theta > 0$
70. $\sin \theta = -\frac{2}{4}$	$\cos \theta > 0$

In Exercises 71–74, find the reference angle  $\theta'$ , and sketch  $\theta$  and  $\theta'$  in standard position.

- |                                |                                |
|--------------------------------|--------------------------------|
| 71. $\theta = 264^\circ$       | 72. $\theta = 635^\circ$       |
| 73. $\theta = -\frac{6\pi}{5}$ | 74. $\theta = \frac{17\pi}{3}$ |

In Exercises 75–82, evaluate the sine, cosine, and tangent of the angle without using a calculator.

- |                       |                       |
|-----------------------|-----------------------|
| 75. $\frac{\pi}{3}$   | 76. $\frac{\pi}{4}$   |
| 77. $-\frac{7\pi}{3}$ | 78. $-\frac{5\pi}{4}$ |
| 79. $495^\circ$       | 80. $-150^\circ$      |
| 81. $-240^\circ$      | 82. $315^\circ$       |



In Exercises 83–88, use a calculator to evaluate the trigonometric function. Round your answer to four decimal places.

- |                            |   |
|----------------------------|---|
| 83. $\sin 4$               | 84. $\tan 3$                            |
| 85. $\sin(-3.2)$           | 86. $\cot(-4.8)$                        |
| 85. $\sin \frac{12\pi}{5}$ | 88. $\tan\left(-\frac{25\pi}{7}\right)$ |

**4.5** In Exercises 89–96, sketch the graph of the function. Include two full periods.

- |  |  |
|--|--|
| 89. $y = \sin x$                       | 90. $y = \cos x$                             |
| 91. $f(x) = 5 \sin \frac{2x}{5}$       | 92. $f(x) = 8 \cos\left(-\frac{x}{4}\right)$ |
| 93. $y = 2 + \sin x$                   | 94. $y = -4 - \cos \pi x$                    |
| 95. $g(t) = \frac{5}{2} \sin(t - \pi)$ | 96. $g(t) = 3 \cos(t + \pi)$                 |

97. **Sound Waves** Sound waves can be modeled by sine functions of the form  $y = a \sin bx$ , where  $x$  is measured in seconds.

- (a) Write an equation of a sound wave whose amplitude is 2 and whose period is  $\frac{1}{264}$  second.  
 (b) What is the frequency of the sound wave described in part (a)?

**98. Data Analysis: Meteorology** The times  $S$  of sunset (Greenwich Mean Time) at  $40^\circ$  north latitude on the 15th of each month are: 1(16:59), 2(17:35), 3(18:06), 4(18:38), 5(19:08), 6(19:30), 7(19:28), 8(18:57), 9(18:09), 10(17:21), 11(16:44), 12(16:36). The month is represented by  $t$ , with  $t = 1$  corresponding to January. A model (in which minutes have been converted to the decimal parts of an hour) for the data is

$$S(t) = 18.09 + 1.41 \sin\left(\frac{\pi t}{6} + 4.60\right).$$

- (a) Use a graphing utility to graph the data points and the model in the same viewing window.
- (b) What is the period of the model? Is it what you expected? Explain.
- (c) What is the amplitude of the model? What does it represent in the model? Explain.

**4.6** In Exercises 99–106, sketch a graph of the function. Include two full periods.

99.  $f(x) = \tan x$       100.  $f(t) = \tan\left(t - \frac{\pi}{4}\right)$
101.  $f(x) = \cot x$
102.  $g(t) = 2 \cot 2t$
103.  $f(x) = \sec x$
104.  $h(t) = \sec\left(t - \frac{\pi}{4}\right)$
105.  $f(x) = \csc x$
106.  $f(t) = 3 \csc\left(2t + \frac{\pi}{4}\right)$

In Exercises 107 and 108, use a graphing utility to graph the function and the damping factor of the function in the same viewing window. Describe the behavior of the function as  $x$  increases without bound.

107.  $f(x) = x \cos x$       108.  $g(x) = x^4 \cos x$

**4.7** In Exercises 109–114, evaluate the expression. If necessary, round your answer to two decimal places.

109.  $\arcsin\left(-\frac{1}{2}\right)$       110.  $\arcsin(-1)$
111.  $\arcsin 0.4$       112.  $\arcsin 0.213$
113.  $\sin^{-1}(-0.44)$       114.  $\sin^{-1} 0.89$

In Exercises 115–118, evaluate the expression without the aid of a calculator.

115.  $\arccos \frac{\sqrt{3}}{2}$       116.  $\arccos \frac{\sqrt{2}}{2}$
117.  $\cos^{-1}(-1)$       118.  $\cos^{-1} \frac{\sqrt{3}}{2}$

In Exercises 119–122, use a calculator to evaluate the expression. Round your answer to two decimal places.

119.  $\arccos 0.324$       120.  $\arccos(-0.888)$
121.  $\tan^{-1}(-1.5)$       122.  $\tan^{-1} 8.2$

In Exercises 123–126, use a graphing utility to graph the function.

123.  $f(x) = 2 \arcsin x$       124.  $f(x) = 3 \arccos x$
125.  $f(x) = \arctan \frac{x}{2}$
126.  $f(x) = -\arcsin 2x$

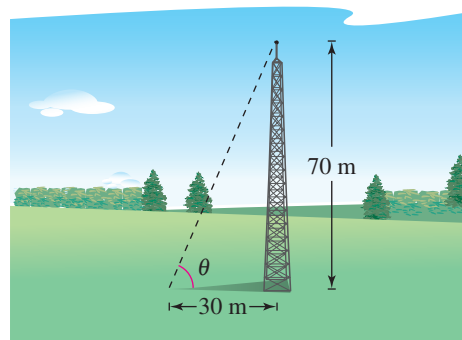
In Exercises 127–130, find the exact value of the expression.

127.  $\cos\left(\arctan \frac{3}{4}\right)$
128.  $\tan\left(\arccos \frac{3}{5}\right)$
129.  $\sec\left(\arctan \frac{12}{5}\right)$
130.  $\cot\left[\arcsin\left(-\frac{12}{13}\right)\right]$

In Exercises 131 and 132, write an algebraic expression that is equivalent to the expression.

131.  $\tan\left(\arccos \frac{x}{2}\right)$
132.  $\sec[\arcsin(x - 1)]$

**4.8** **133. Angle of Elevation** The height of a radio transmission tower is 70 meters, and it casts a shadow of length 30 meters (see figure). Find the angle of elevation of the sun.



**134. Height** Your football has landed at the edge of the roof of your school building. When you are 25 feet from the base of the building, the angle of elevation to your football is  $21^\circ$ . How high off the ground is your football?

**135. Distance** From city  $A$  to city  $B$ , a plane flies 650 miles at a bearing of  $48^\circ$ . From city  $B$  to city  $C$ , the plane flies 810 miles at a bearing of  $115^\circ$ . Find the distance from city  $A$  to city  $C$  and the bearing from city  $A$  to city  $C$ .

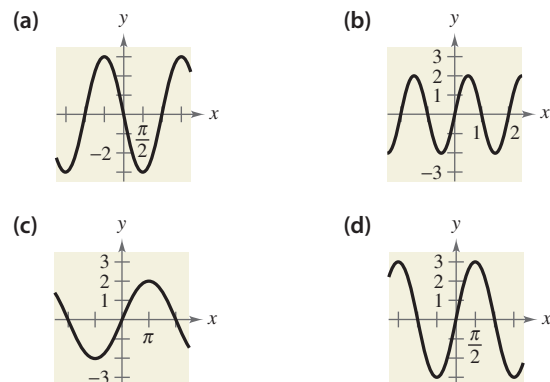
- 136. Wave Motion** Your fishing bobber oscillates in simple harmonic motion from the waves in the lake where you fish. Your bobber moves a total of 1.5 inches from its high point to its low point and returns to its high point every 3 seconds. Write an equation modeling the motion of your bobber if it is at its high point at time  $t = 0$ .

### Synthesis

**True or False?** In Exercises 137–140, determine whether the statement is true or false. Justify your answer.

- 137.** The tangent function is often useful for modeling simple harmonic motion.  
**138.** The inverse sine function  $y = \arcsin x$  cannot be defined as a function over any interval that is greater than the interval defined as  $-\pi/2 \leq y \leq \pi/2$ .  
**139.**  $y = \sin \theta$  is not a function because  $\sin 30^\circ = \sin 150^\circ$ .  
**140.** Because  $\tan 3\pi/4 = -1$ ,  $\arctan(-1) = 3\pi/4$ .

In Exercises 141–144, match the function  $y = a \sin bx$  with its graph. Base your selection solely on your interpretation of the constants  $a$  and  $b$ . Explain your reasoning. [The graphs are labeled (a), (b), (c), and (d).]



- 141.**  $y = 3 \sin x$                       **142.**  $y = -3 \sin x$   
**143.**  $y = 2 \sin \pi x$                       **144.**  $y = 2 \sin \frac{x}{2}$

- 145. Writing** Describe the behavior of  $f(\theta) = \sec \theta$  at the zeros of  $g(\theta) = \cos \theta$ . Explain your reasoning.

### 146. Conjecture

- (a) Use a graphing utility to complete the table.

$\theta$	0.1	0.4	0.7	1.0	1.3
$\tan\left(\theta - \frac{\pi}{2}\right)$					
$-\cot \theta$					

- (b) Make a conjecture about the relationship between  $\tan\left(\theta - \frac{\pi}{2}\right)$  and  $-\cot \theta$ .

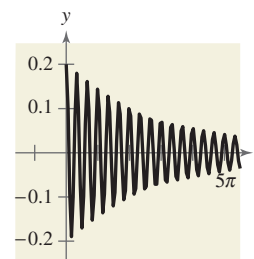
- 147. Writing** When graphing the sine and cosine functions, determining the amplitude is part of the analysis. Explain why this is not true for the other four trigonometric functions.

- 148. Oscillation of a Spring** A weight is suspended from a ceiling by a steel spring. The weight is lifted (positive direction) from the equilibrium position and released. The resulting motion of the weight is modeled by

$$y = Ae^{-kt} \cos bt = \frac{1}{5}e^{-t/10} \cos 6t$$

where  $y$  is the distance in feet from equilibrium and  $t$  is the time in seconds. The graph of the function is shown in the figure. For each of the following, describe the change in the system without graphing the resulting function.

- (a)  $A$  is changed from  $\frac{1}{5}$  to  $\frac{1}{3}$ .  
 (b)  $k$  is changed from  $\frac{1}{10}$  to  $\frac{1}{3}$ .  
 (c)  $b$  is changed from 6 to 9.



- 149. Graphical Reasoning** The formulas for the area of a circular sector and arc length are  $A = \frac{1}{2}r^2\theta$  and  $s = r\theta$ , respectively. ( $r$  is the radius and  $\theta$  is the angle measured in radians.)

- (a) For  $\theta = 0.8$ , write the area and arc length as functions of  $r$ . What is the domain of each function? Use a graphing utility to graph the functions. Use the graphs to determine which function changes more rapidly as  $r$  increases. Explain.  
 (b) For  $r = 10$  centimeters, write the area and arc length as functions of  $\theta$ . What is the domain of each function? Use a graphing utility to graph and identify the functions.

- 150. Writing** Describe a real-life application that can be represented by a simple harmonic motion model and is different from any that you've seen in this chapter. Explain which function you would use to model your application and why. Explain how you would determine the amplitude, period, and frequency of the model for your application.

## 4 Chapter Test

Take this test as you would take a test in class. When you are finished, check your work against the answers given in the back of the book.

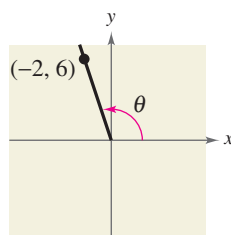


FIGURE FOR 4

- Consider an angle that measures  $\frac{5\pi}{4}$  radians.
  - Sketch the angle in standard position.
  - Determine two coterminal angles (one positive and one negative).
  - Convert the angle to degree measure.
- A truck is moving at a rate of 90 kilometers per hour, and the diameter of its wheels is 1 meter. Find the angular speed of the wheels in radians per minute.
- A water sprinkler sprays water on a lawn over a distance of 25 feet and rotates through an angle of  $130^\circ$ . Find the area of the lawn watered by the sprinkler.
- Find the exact values of the six trigonometric functions of the angle  $\theta$  shown in the figure.
- Given that  $\tan \theta = \frac{3}{2}$ , find the other five trigonometric functions of  $\theta$ .
- Determine the reference angle  $\theta'$  of the angle  $\theta = 290^\circ$  and sketch  $\theta$  and  $\theta'$  in standard position.
- Determine the quadrant in which  $\theta$  lies if  $\sec \theta < 0$  and  $\tan \theta > 0$ .
- Find two exact values of  $\theta$  in degrees ( $0 \leq \theta < 360^\circ$ ) if  $\cos \theta = -\sqrt{3}/2$ . (Do not use a calculator.)
- Use a calculator to approximate two values of  $\theta$  in radians ( $0 \leq \theta < 2\pi$ ) if  $\csc \theta = 1.030$ . Round the results to two decimal places.

In Exercises 10 and 11, find the remaining five trigonometric functions of  $\theta$  satisfying the conditions.

10.  $\cos \theta = \frac{3}{5}$ ,  $\tan \theta < 0$

11.  $\sec \theta = -\frac{17}{8}$ ,  $\sin \theta > 0$

In Exercises 12 and 13, sketch the graph of the function. (Include two full periods.)

12.  $g(x) = -2 \sin\left(x - \frac{\pi}{4}\right)$

13.  $f(\alpha) = \frac{1}{2} \tan 2\alpha$

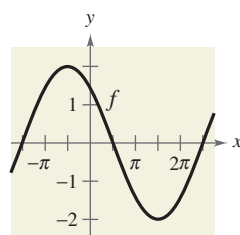


FIGURE FOR 16

In Exercises 14 and 15, use a graphing utility to graph the function. If the function is periodic, find its period.

14.  $y = \sin 2\pi x + 2 \cos \pi x$

15.  $y = 6e^{-0.12t} \cos(0.25t)$ ,  $0 \leq t \leq 32$

- Find  $a$ ,  $b$ , and  $c$  for the function  $f(x) = a \sin(bx + c)$  such that the graph of  $f$  matches the figure.
- Find the exact value of  $\tan(\arccos \frac{2}{3})$  without the aid of a calculator.
- Graph the function  $f(x) = 2 \arcsin(\frac{1}{2}x)$ .
- A plane is 80 miles south and 95 miles east of Cleveland Hopkins International Airport. What bearing should be taken to fly directly to the airport?
- Write the equation for the simple harmonic motion of a ball on a spring that starts at its lowest point of 6 inches below equilibrium, bounces to its maximum height of 6 inches above equilibrium, and returns to its lowest point in a total of 2 seconds.

# Proofs in Mathematics

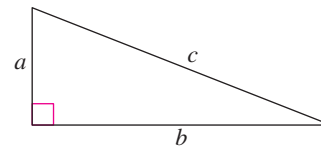
## The Pythagorean Theorem

The Pythagorean Theorem is one of the most famous theorems in mathematics. More than 100 different proofs now exist. James A. Garfield, the twentieth president of the United States, developed a proof of the Pythagorean Theorem in 1876. His proof, shown below, involved the fact that a trapezoid can be formed from two congruent right triangles and an isosceles right triangle.

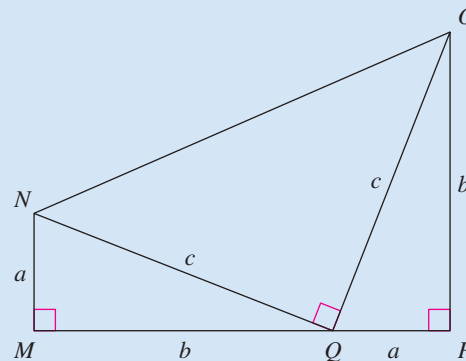
### The Pythagorean Theorem

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse, where  $a$  and  $b$  are the legs and  $c$  is the hypotenuse.

$$a^2 + b^2 = c^2$$



### Proof



$$\text{Area of trapezoid } MNOP = \text{Area of } \triangle MNQ + \text{Area of } \triangle PQO + \text{Area of } \triangle NOQ$$

$$\frac{1}{2}(a + b)(a + b) = \frac{1}{2}ab + \frac{1}{2}ab + \frac{1}{2}c^2$$

$$\frac{1}{2}(a + b)(a + b) = ab + \frac{1}{2}c^2$$

$$(a + b)(a + b) = 2ab + c^2$$

$$a^2 + 2ab + b^2 = 2ab + c^2$$

$$a^2 + b^2 = c^2$$

## P.S.

## Problem Solving

This collection of thought-provoking and challenging exercises further explores and expands upon concepts learned in this chapter.

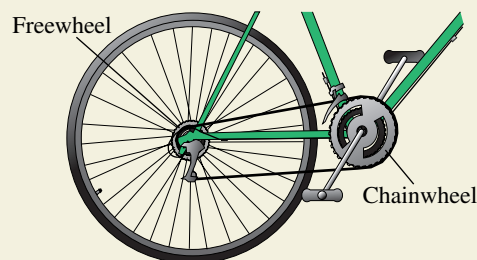
1. The restaurant at the top of the Space Needle in Seattle, Washington is circular and has a radius of 47.25 feet. The dining part of the restaurant revolves, making about one complete revolution every 48 minutes. A dinner party was seated at the edge of the revolving restaurant at 6:45 P.M. and was finished at 8:57 P.M.

- (a) Find the angle through which the dinner party rotated.  
(b) Find the distance the party traveled during dinner.

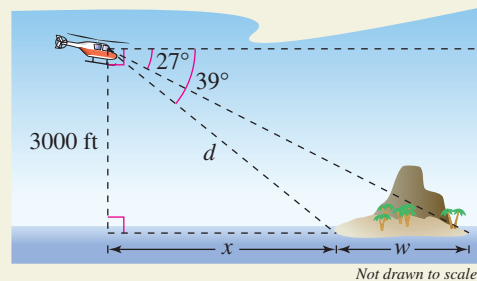
2. A bicycle's gear ratio is the number of times the freewheel turns for every one turn of the chainwheel (see figure). The table shows the numbers of teeth in the freewheel and chainwheel for the first five gears of an 18-speed touring bicycle. The chainwheel completes one rotation for each gear. Find the angle through which the freewheel turns for each gear. Give your answers in both degrees and radians.



Gear number	Number of teeth in freewheel	Number of teeth in chainwheel
1	32	24
2	26	24
3	22	24
4	32	40
5	19	24

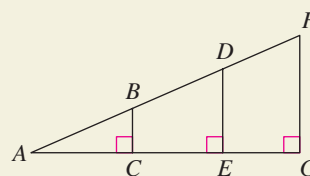


3. A surveyor in a helicopter is trying to determine the width of an island, as shown in the figure.



- (a) What is the shortest distance  $d$  the helicopter would have to travel to land on the island?  
(b) What is the horizontal distance  $x$  that the helicopter would have to travel before it would be directly over the nearer end of the island?  
(c) Find the width  $w$  of the island. Explain how you obtained your answer.

4. Use the figure below.



- (a) Explain why  $\triangle ABC$ ,  $\triangle ADE$ , and  $\triangle AFG$  are similar triangles.  
(b) What does similarity imply about the ratios

$$\frac{BC}{AB}, \frac{DE}{AD}, \text{ and } \frac{FG}{AF}?$$

- (c) Does the value of  $\sin A$  depend on which triangle from part (a) is used to calculate it? Would the value of  $\sin A$  change if it were found using a different right triangle that was similar to the three given triangles?  
(d) Do your conclusions from part (c) apply to the other five trigonometric functions? Explain.



5. Use a graphing utility to graph  $h$ , and use the graph to decide whether  $h$  is even, odd, or neither.

- (a)  $h(x) = \cos^2 x$   
(b)  $h(x) = \sin^2 x$

6. If  $f$  is an even function and  $g$  is an odd function, use the results of Exercise 5 to make a conjecture about  $h$ , where

- (a)  $h(x) = [f(x)]^2$   
(b)  $h(x) = [g(x)]^2$ .

7. The model for the height  $h$  (in feet) of a Ferris wheel car is

$$h = 50 + 50 \sin 8\pi t$$

where  $t$  is the time (in minutes). (The Ferris wheel has a radius of 50 feet.) This model yields a height of 50 feet when  $t = 0$ . Alter the model so that the height of the car is 1 foot when  $t = 0$ .



8. The pressure  $P$  (in millimeters of mercury) against the walls of the blood vessels of a patient is modeled by

$$P = 100 - 20 \cos\left(\frac{8\pi}{3}t\right)$$

where  $t$  is time (in seconds).



- Use a graphing utility to graph the model.
  - What is the period of the model? What does the period tell you about this situation?
  - What is the amplitude of the model? What does it tell you about this situation?
  - If one cycle of this model is equivalent to one heart-beat, what is the pulse of this patient?
  - If a physician wants this patient's pulse rate to be 64 beats per minute or less, what should the period be? What should the coefficient of  $t$  be?
9. A popular theory that attempts to explain the ups and downs of everyday life states that each of us has three cycles, called biorhythms, which begin at birth. These three cycles can be modeled by sine waves.

Physical (23 days):  $P = \sin \frac{2\pi t}{23}, \quad t \geq 0$

Emotional (28 days):  $E = \sin \frac{2\pi t}{28}, \quad t \geq 0$

Intellectual (33 days):  $I = \sin \frac{2\pi t}{33}, \quad t \geq 0$

where  $t$  is the number of days since birth. Consider a person who was born on July 20, 1986.



- Use a graphing utility to graph the three models in the same viewing window for  $7300 \leq t \leq 7380$ .
- Describe the person's biorhythms during the month of September 2006.
- Calculate the person's three energy levels on September 22, 2006.



10. (a) Use a graphing utility to graph the functions given by

$$f(x) = 2 \cos 2x + 3 \sin 3x$$

and

$$g(x) = 2 \cos 2x + 3 \sin 4x.$$

- Use the graphs from part (a) to find the period of each function.
- If  $\alpha$  and  $\beta$  are positive integers, is the function given by  $h(x) = A \cos \alpha x + B \sin \beta x$  periodic? Explain your reasoning.

11. Two trigonometric functions  $f$  and  $g$  have periods of 2, and their graphs intersect at  $x = 5.35$ .

- Give one smaller and one larger positive value of  $x$  at which the functions have the same value.
- Determine one negative value of  $x$  at which the graphs intersect.
- Is it true that  $f(13.35) = g(-4.65)$ ? Explain your reasoning.

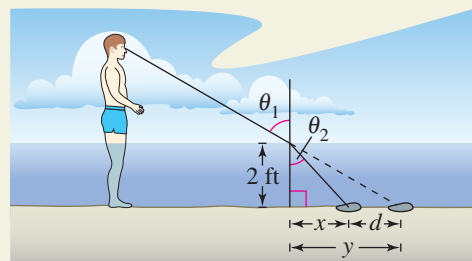
12. The function  $f$  is periodic, with period  $c$ . So,  $f(t + c) = f(t)$ . Are the following equal? Explain.

(a)  $f(t - 2c) = f(t)$

(b)  $f\left(t + \frac{1}{2}c\right) = f\left(\frac{1}{2}t\right)$

(c)  $f\left(\frac{1}{2}(t + c)\right) = f\left(\frac{1}{2}t\right)$

13. If you stand in shallow water and look at an object below the surface of the water, the object will look farther away from you than it really is. This is because when light rays pass between air and water, the water refracts, or bends, the light rays. The index of refraction for water is 1.333. This is the ratio of the sine of  $\theta_1$  and the sine of  $\theta_2$  (see figure).



- You are standing in water that is 2 feet deep and are looking at a rock at angle  $\theta_1 = 60^\circ$  (measured from a line perpendicular to the surface of the water). Find  $\theta_2$ .
- Find the distances  $x$  and  $y$ .
- Find the distance  $d$  between where the rock is and where it appears to be.
- What happens to  $d$  as you move closer to the rock? Explain your reasoning.



14. In calculus, it can be shown that the arctangent function can be approximated by the polynomial

$$\arctan x \approx x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7}$$

where  $x$  is in radians.

- Use a graphing utility to graph the arctangent function and its polynomial approximation in the same viewing window. How do the graphs compare?
- Study the pattern in the polynomial approximation of the arctangent function and guess the next term. Then repeat part (a). How does the accuracy of the approximation change when additional terms are added?