4.6 Graphs of Other Trigonometric Functions

What you should learn

- Sketch the graphs of tangent functions.
- Sketch the graphs of cotangent functions.
- Sketch the graphs of secant and cosecant functions.
- Sketch the graphs of damped trigonometric functions.

Why you should learn it

Trigonometric functions can be used to model real-life situations such as the distance from a television camera to a unit in a parade as in Exercise 76 on page 341.

Photodisc/Getty Images

Graph of the Tangent Function

Recall that the tangent function is odd. That is, $\tan(-x) = -\tan x$. Consequently, the graph of $y = \tan x$ is symmetric with respect to the origin. You also know from the identity $\tan x = \sin x/\cos x$ that the tangent is undefined for values at which $\cos x = 0$. Two such values are $x = \pm\pi/2 \approx \pm 1.5708$.

x	$-\frac{\pi}{2}$	-1.57	-1.5	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	1.5	1.57	$\frac{\pi}{2}$
$\tan x$	Undef.	-1255.8	-14.1	-1	0	1	14.1	1255.8	Undef.

As indicated in the table, $\tan x$ increases without bound as x approaches $\pi/2$ from the left, and decreases without bound as x approaches $-\pi/2$ from the right. So, the graph of $y = \tan x$ has *vertical asymptotes* at $x = \pi/2$ and $x = -\pi/2$, as shown in Figure 4.59. Moreover, because the period of the tangent function is π, vertical asymptotes also occur when $x = \pi/2 + n\pi$, where n is an integer. The domain of the tangent function is the set of all real numbers other than $x = \pi/2 + n\pi$, and the range is the set of all real numbers.

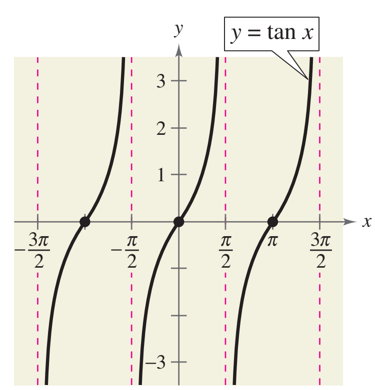


FIGURE 4.59

PERIOD: π
DOMAIN: ALL $x \neq \frac{\pi}{2} + n\pi$
RANGE: $(-\infty, \infty)$
VERTICAL ASYMPTOTES: $x = \frac{\pi}{2} + n\pi$

Sketching the graph of $y = a\tan(bx - c)$ is similar to sketching the graph of $y = a\sin(bx - c)$ in that you locate key points that identify the intercepts and asymptotes. Two consecutive vertical asymptotes can be found by solving the equations

$$bx - c = -\frac{\pi}{2} \quad \text{and} \quad bx - c = \frac{\pi}{2}.$$

The midpoint between two consecutive vertical asymptotes is an x-intercept of the graph. The period of the function $y = a\tan(bx - c)$ is the distance between two consecutive vertical asymptotes. The amplitude of a tangent function is not defined. After plotting the asymptotes and the x-intercept, plot a few additional points between the two asymptotes and sketch one cycle. Finally, sketch one or two additional cycles to the left and right.

Consider reviewing period, range, and domain for all six trigonometric functions, especially emphasizing the difference between the periods of the tangent and cotangent functions.

Example 1 Sketching the Graph of a Tangent Function

Sketch the graph of $y = \tan \frac{x}{2}$.

Solution

By solving the equations

$$\frac{x}{2} = -\frac{\pi}{2} \quad \text{and} \quad \frac{x}{2} = \frac{\pi}{2}$$

$$x = -\pi \qquad\qquad x = \pi$$

you can see that two consecutive vertical asymptotes occur at $x = -\pi$ and $x = \pi$. Between these two asymptotes, plot a few points, including the x-intercept, as shown in the table. Three cycles of the graph are shown in Figure 4.60.

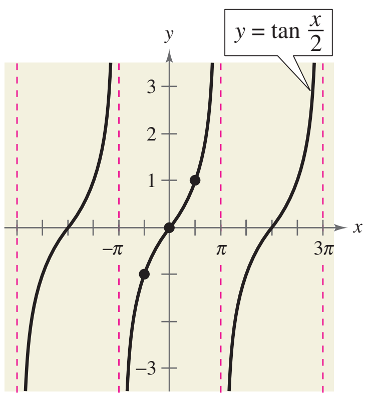


FIGURE 4.60

x	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π
$\tan \frac{x}{2}$	Undef.	-1	0	1	Undef.

CHECKPOINT Now try Exercise 7.

Example 2 Sketching the Graph of a Tangent Function

Sketch the graph of $y = -3 \tan 2x$.

Solution

By solving the equations

$$2x = -\frac{\pi}{2} \quad \text{and} \quad 2x = \frac{\pi}{2}$$

$$x = -\frac{\pi}{4} \qquad\qquad x = \frac{\pi}{4}$$

you can see that two consecutive vertical asymptotes occur at $x = -\pi/4$ and $x = \pi/4$. Between these two asymptotes, plot a few points, including the x-intercept, as shown in the table. Three cycles of the graph are shown in Figure 4.61.

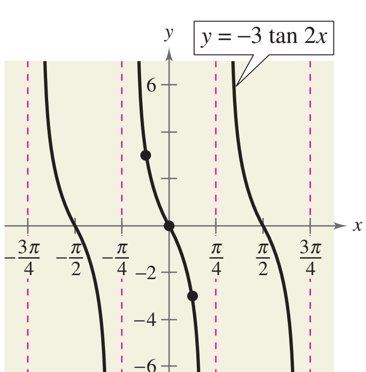


FIGURE 4.61

x	$-\frac{\pi}{4}$	$-\frac{\pi}{8}$	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$
$-3 \tan 2x$	Undef.	3	0	-3	Undef.

CHECKPOINT Now try Exercise 9.

By comparing the graphs in Examples 1 and 2, you can see that the graph of $y = a \tan(bx - c)$ increases between consecutive vertical asymptotes when $a > 0$, and decreases between consecutive vertical asymptotes when $a < 0$. In other words, the graph for $a < 0$ is a reflection in the x-axis of the graph for $a > 0$.

Graph of the Cotangent Function

The graph of the cotangent function is similar to the graph of the tangent function. It also has a period of π. However, from the identity

$$y = \cot x = \frac{\cos x}{\sin x}$$

you can see that the cotangent function has vertical asymptotes when $\sin x$ is zero, which occurs at $x = n\pi$, where n is an integer. The graph of the cotangent function is shown in Figure 4.62. Note that two consecutive vertical asymptotes of the graph of $y = a \cot(bx - c)$ can be found by solving the equations $bx - c = 0$ and $bx - c = \pi$.

Technology

Some graphing utilities have difficulty graphing trigonometric functions that have vertical asymptotes. Your graphing utility may connect parts of the graphs of tangent, cotangent, secant, and cosecant functions that are not supposed to be connected. To eliminate this problem, change the mode of the graphing utility to *dot* mode.

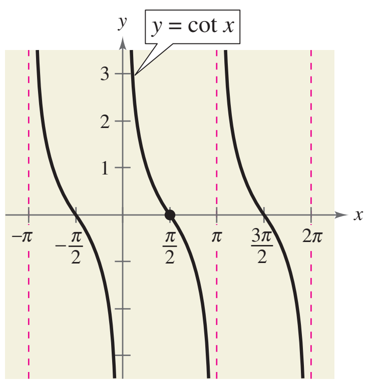


FIGURE 4.62

PERIOD: π
DOMAIN: ALL $x \neq n\pi$
RANGE: $(-\infty, \infty)$
VERTICAL ASYMPTOTES: $x = n\pi$

Example 3 Sketching the Graph of a Cotangent Function

Sketch the graph of $y = 2 \cot \frac{x}{3}$.

Solution

By solving the equations

$$\frac{x}{3} = 0 \qquad \text{and} \qquad \frac{x}{3} = \pi$$

$$x = 0 \qquad\qquad x = 3\pi$$

you can see that two consecutive vertical asymptotes occur at $x = 0$ and $x = 3\pi$. Between these two asymptotes, plot a few points, including the x-intercept, as shown in the table. Three cycles of the graph are shown in Figure 4.63. Note that the period is 3π, the distance between consecutive asymptotes.

x	0	$\frac{3\pi}{4}$	$\frac{3\pi}{2}$	$\frac{9\pi}{4}$	3π
$2 \cot \frac{x}{3}$	Undef.	2	0	-2	Undef.

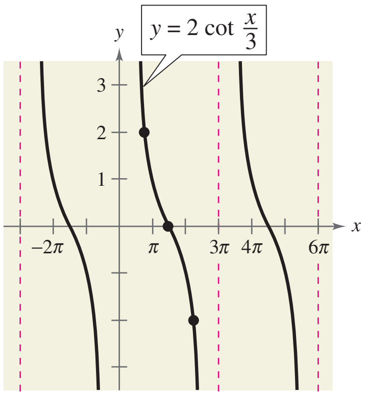


FIGURE 4.63

CHECKPOINT Now try Exercise 19.

Graphs of the Reciprocal Functions

Using a graphing utility to graph cosecant and secant functions can help reinforce the need to use the reciprocal functions.

The graphs of the two remaining trigonometric functions can be obtained from the graphs of the sine and cosine functions using the reciprocal identities

$$\csc x = \frac{1}{\sin x} \qquad \text{and} \qquad \sec x = \frac{1}{\cos x}.$$

For instance, at a given value of x, the y-coordinate of $\sec x$ is the reciprocal of the y-coordinate of $\cos x$. Of course, when $\cos x = 0$, the reciprocal does not exist. Near such values of x, the behavior of the secant function is similar to that of the tangent function. In other words, the graphs of

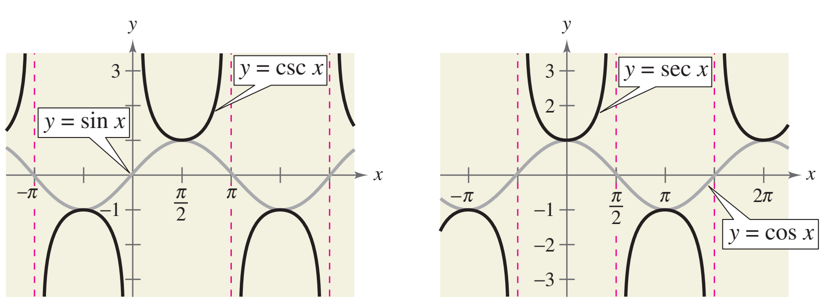
$$\tan x = \frac{\sin x}{\cos x} \qquad \text{and} \qquad \sec x = \frac{1}{\cos x}$$

have vertical asymptotes at $x = \pi/2 + n\pi$, where n is an integer, and the cosine is zero at these x-values. Similarly,

$$\cot x = \frac{\cos x}{\sin x} \qquad \text{and} \qquad \csc x = \frac{1}{\sin x}$$

have vertical asymptotes where $\sin x = 0$—that is, at $x = n\pi$.

To sketch the graph of a secant or cosecant function, you should first make a sketch of its reciprocal function. For instance, to sketch the graph of $y = \csc x$, first sketch the graph of $y = \sin x$. Then take reciprocals of the y-coordinates to obtain points on the graph of $y = \csc x$. This procedure is used to obtain the graphs shown in Figure 4.64.



PERIOD: 2π
DOMAIN: ALL $x \neq n\pi$
RANGE: $(-\infty, -1] \cup [1, \infty)$
VERTICAL ASYMPTOTES: $x = n\pi$
SYMMETRY: ORIGIN

PERIOD: 2π
DOMAIN: ALL $x \neq \frac{\pi}{2} + n\pi$
RANGE: $(-\infty, -1] \cup [1, \infty)$
VERTICAL ASYMPTOTES: $x = \frac{\pi}{2} + n\pi$
SYMMETRY: y-AXIS

FIGURE 4.64

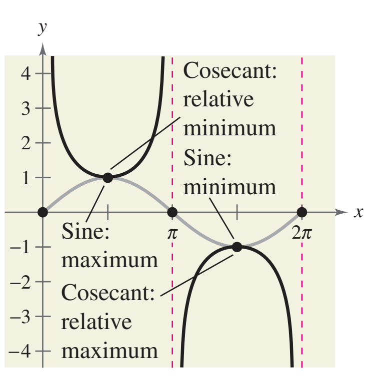


FIGURE 4.65

In comparing the graphs of the cosecant and secant functions with those of the sine and cosine functions, note that the "hills" and "valleys" are interchanged. For example, a hill (or maximum point) on the sine curve corresponds to a valley (a relative minimum) on the cosecant curve, and a valley (or minimum point) on the sine curve corresponds to a hill (a relative maximum) on the cosecant curve, as shown in Figure 4.65. Additionally, x-intercepts of the sine and cosine functions become vertical asymptotes of the cosecant and secant functions, respectively (see Figure 4.65).

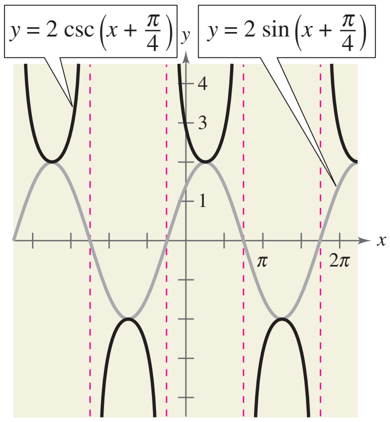


FIGURE 4.66

Example 4 Sketching the Graph of a Cosecant Function

Sketch the graph of $y = 2\csc\left(x + \frac{\pi}{4}\right)$.

Solution

Begin by sketching the graph of

$$y = 2\sin\left(x + \frac{\pi}{4}\right).$$

For this function, the amplitude is 2 and the period is 2π. By solving the equations

$$x + \frac{\pi}{4} = 0 \qquad \text{and} \qquad x + \frac{\pi}{4} = 2\pi$$

$$x = -\frac{\pi}{4} \qquad\qquad x = \frac{7\pi}{4}$$

you can see that one cycle of the sine function corresponds to the interval from $x = -\pi/4$ to $x = 7\pi/4$. The graph of this sine function is represented by the gray curve in Figure 4.66. Because the sine function is zero at the midpoint and endpoints of this interval, the corresponding cosecant function

$$\begin{aligned} y &= 2\csc\left(x + \frac{\pi}{4}\right) \\ &= 2\left(\frac{1}{\sin[x + (\pi/4)]}\right) \end{aligned}$$

has vertical asymptotes at $x = -\pi/4, x = 3\pi/4, x = 7\pi/4$, etc. The graph of the cosecant function is represented by the black curve in Figure 4.66.

CHECKPOINT Now try Exercise 25.

Example 5 Sketching the Graph of a Secant Function

Sketch the graph of $y = \sec 2x$.

Solution

Begin by sketching the graph of $y = \cos 2x$, as indicated by the gray curve in Figure 4.67. Then, form the graph of $y = \sec 2x$ as the black curve in the figure. Note that the x-intercepts of $y = \cos 2x$

$$\left(-\frac{\pi}{4}, 0\right), \quad \left(\frac{\pi}{4}, 0\right), \quad \left(\frac{3\pi}{4}, 0\right), \ldots$$

correspond to the vertical asymptotes

$$x = -\frac{\pi}{4}, \quad x = \frac{\pi}{4}, \quad x = \frac{3\pi}{4}, \ldots$$

of the graph of $y = \sec 2x$. Moreover, notice that the period of $y = \cos 2x$ and $y = \sec 2x$ is π.

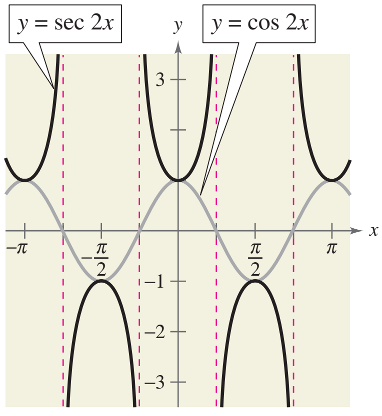


FIGURE 4.67

CHECKPOINT Now try Exercise 27.

Damped Trigonometric Graphs

A *product* of two functions can be graphed using properties of the individual functions. For instance, consider the function

$$f(x) = x \sin x$$

as the product of the functions $y = x$ and $y = \sin x$. Using properties of absolute value and the fact that $|\sin x| \le 1$, you have $0 \le |x||\sin x| \le |x|$. Consequently,

$$-|x| \le x \sin x \le |x|$$

which means that the graph of $f(x) = x \sin x$ lies between the lines $y = -x$ and $y = x$. Furthermore, because

$$f(x) = x \sin x = \pm x \quad \text{at} \quad x = \frac{\pi}{2} + n\pi$$

and

$$f(x) = x \sin x = 0 \quad \text{at} \quad x = n\pi$$

the graph of f touches the line $y = -x$ or the line $y = x$ at $x = \pi/2 + n\pi$ and has x-intercepts at $x = n\pi$. A sketch of f is shown in Figure 4.68. In the function $f(x) = x \sin x$, the factor x is called the **damping factor.**

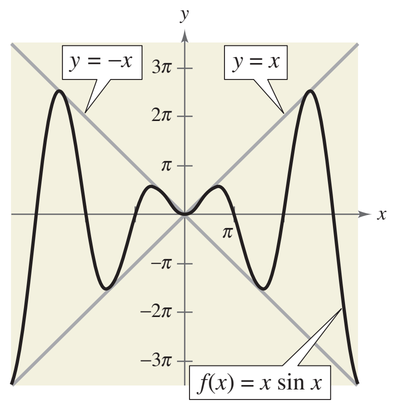


FIGURE 4.68

STUDY TIP

Do you see why the graph of $f(x) = x \sin x$ touches the lines $y = \pm x$ at $x = \pi/2 + n\pi$ and why the graph has x-intercepts at $x = n\pi$? Recall that the sine function is equal to 1 at $\pi/2$, $3\pi/2$, $5\pi/2$, . . . (odd multiples of $\pi/2$) and is equal to 0 at π, 2π, 3π, . . . (multiples of π).

Example 6 Damped Sine Wave

Sketch the graph of

$$f(x) = e^{-x} \sin 3x.$$

Solution

Consider $f(x)$ as the product of the two functions

$$y = e^{-x} \quad \text{and} \quad y = \sin 3x$$

each of which has the set of real numbers as its domain. For any real number x, you know that $e^{-x} \ge 0$ and $|\sin 3x| \le 1$. So, $e^{-x}|\sin 3x| \le e^{-x}$, which means that

$$-e^{-x} \le e^{-x} \sin 3x \le e^{-x}.$$

Furthermore, because

$$f(x) = e^{-x} \sin 3x = \pm e^{-x} \quad \text{at} \quad x = \frac{\pi}{6} + \frac{n\pi}{3}$$

and

$$f(x) = e^{-x} \sin 3x = 0 \quad \text{at} \quad x = \frac{n\pi}{3}$$

the graph of f touches the curves $y = -e^{-x}$ and $y = e^{-x}$ at $x = \pi/6 + n\pi/3$ and has intercepts at $x = n\pi/3$. A sketch is shown in Figure 4.69.

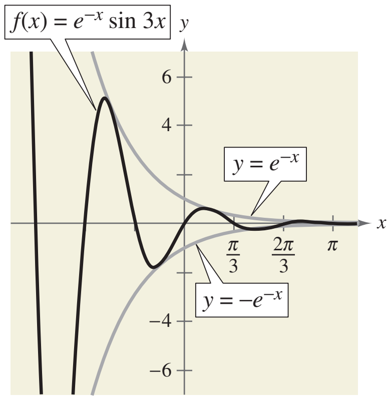
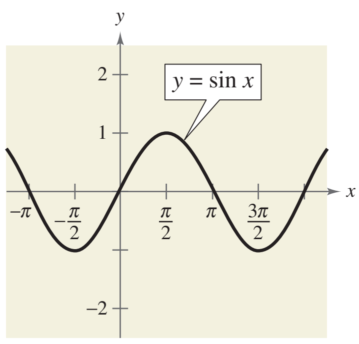


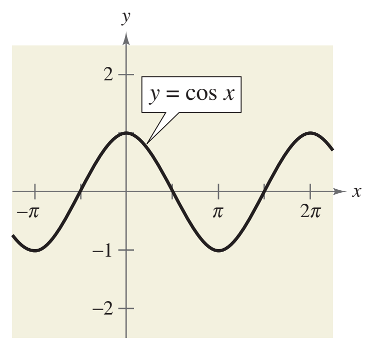
FIGURE 4.69

CHECKPOINT Now try Exercise 65.

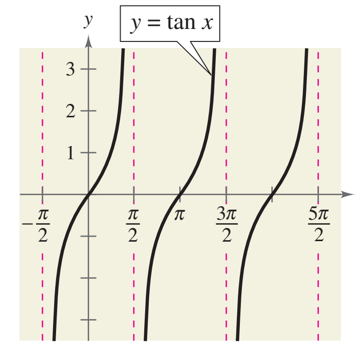
Figure 4.70 summarizes the characteristics of the six basic trigonometric functions.



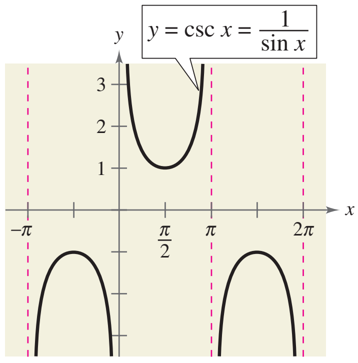
DOMAIN: ALL REALS
RANGE: $[-1, 1]$
PERIOD: 2π



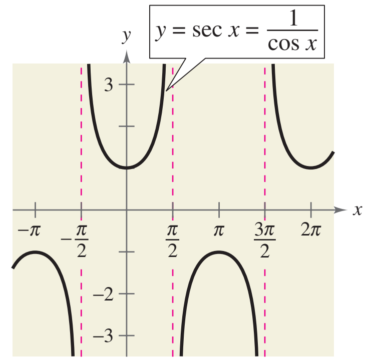
DOMAIN: ALL REALS
RANGE: $[-1, 1]$
PERIOD: 2π



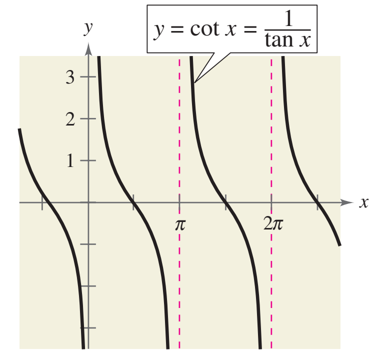
DOMAIN: ALL $x \neq \frac{\pi}{2} + n\pi$
RANGE: $(-\infty, \infty)$
PERIOD: π



DOMAIN: ALL $x \neq n\pi$
RANGE: $(-\infty, -1] \cup [1, \infty)$
PERIOD: 2π



DOMAIN: ALL $x \neq \frac{\pi}{2} + n\pi$
RANGE: $(-\infty, -1] \cup [1, \infty)$
PERIOD: 2π



DOMAIN: ALL $x \neq n\pi$
RANGE: $(-\infty, \infty)$
PERIOD: π

FIGURE 4.70

WRITING ABOUT MATHEMATICS

Combining Trigonometric Functions Recall from Section 1.8 that functions can be combined arithmetically. This also applies to trigonometric functions. For each of the functions

$$h(x) = x + \sin x \quad \text{and} \quad h(x) = \cos x - \sin 3x$$

(a) identify two simpler functions f and g that comprise the combination, (b) use a table to show how to obtain the numerical values of $h(x)$ from the numerical values of $f(x)$ and $g(x)$, and (c) use graphs of f and g to show how h may be formed.

Can you find functions

$$f(x) = d + a\sin(bx + c) \quad \text{and} \quad g(x) = d + a\cos(bx + c)$$

such that $f(x) + g(x) = 0$ for all x?

4.6 Exercises

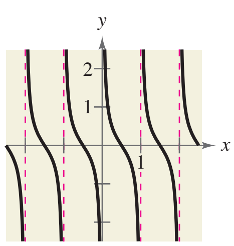
VOCABULARY CHECK: Fill in the blanks.

1. The graphs of the tangent, cotangent, secant, and cosecant functions all have ________ asymptotes.
2. To sketch the graph of a secant or cosecant function, first make a sketch of its corresponding ________ function.
3. For the functions given by $f(x) = g(x) \cdot \sin x$, $g(x)$ is called the ________ factor of the function $f(x)$.
4. The period of $y = \tan x$ is ________.
5. The domain of $y = \cot x$ is all real numbers such that ________.
6. The range of $y = \sec x$ is ________.
7. The period of $y = \csc x$ is ________.

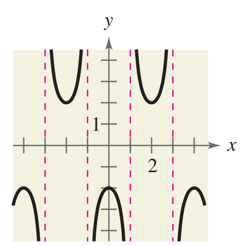
PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

In Exercises 1–6, match the function with its graph. State the period of the function. [The graphs are labeled (a), (b), (c), (d), (e), and (f).]

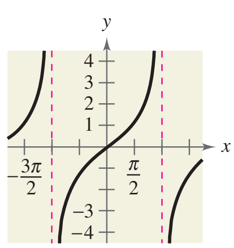
(a)



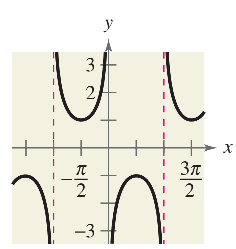
(b)



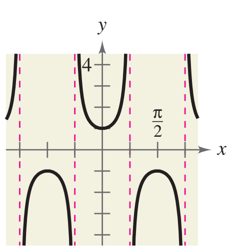
(c)



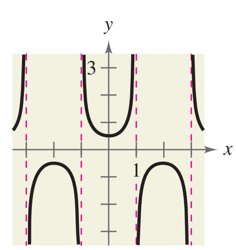
(d)



(e)



(f)



1. $y = \sec 2x$
2. $y = \tan \dfrac{x}{2}$
3. $y = \dfrac{1}{2} \cot \pi x$
4. $y = -\csc x$
5. $y = \dfrac{1}{2} \sec \dfrac{\pi x}{2}$
6. $y = -2 \sec \dfrac{\pi x}{2}$

In Exercises 7–30, sketch the graph of the function. Include two full periods.

7. $y = \frac{1}{3} \tan x$
8. $y = \frac{1}{4} \tan x$
9. $y = \tan 3x$
10. $y = -3 \tan \pi x$
11. $y = -\frac{1}{2} \sec x$
12. $y = \frac{1}{4} \sec x$
13. $y = \csc \pi x$
14. $y = 3 \csc 4x$
15. $y = \sec \pi x - 1$
16. $y = -2 \sec 4x + 2$
17. $y = \csc \dfrac{x}{2}$
18. $y = \csc \dfrac{x}{3}$
19. $y = \cot \dfrac{x}{2}$
20. $y = 3 \cot \dfrac{\pi x}{2}$
21. $y = \frac{1}{2} \sec 2x$
22. $y = -\frac{1}{2} \tan x$
23. $y = \tan \dfrac{\pi x}{4}$
24. $y = \tan(x + \pi)$
25. $y = \csc(\pi - x)$
26. $y = \csc(2x - \pi)$
27. $y = 2 \sec(x + \pi)$
28. $y = -\sec \pi x + 1$
29. $y = \dfrac{1}{4} \csc\left(x + \dfrac{\pi}{4}\right)$
30. $y = 2 \cot\left(x + \dfrac{\pi}{2}\right)$

In Exercises 31–40, use a graphing utility to graph the function. Include two full periods.

31. $y = \tan \dfrac{x}{3}$
32. $y = -\tan 2x$
33. $y = -2 \sec 4x$
34. $y = \sec \pi x$
35. $y = \tan\left(x - \dfrac{\pi}{4}\right)$
36. $y = \dfrac{1}{4} \cot\left(x - \dfrac{\pi}{2}\right)$
37. $y = -\csc(4x - \pi)$
38. $y = 2 \sec(2x - \pi)$
39. $y = 0.1 \tan\left(\dfrac{\pi x}{4} + \dfrac{\pi}{4}\right)$
40. $y = \dfrac{1}{3} \sec\left(\dfrac{\pi x}{2} + \dfrac{\pi}{2}\right)$

In Exercises 41–48, use a graph to solve the equation on the interval $[-2\pi, 2\pi]$.

41. $\tan x = 1$

42. $\tan x = \sqrt{3}$

43. $\cot x = -\dfrac{\sqrt{3}}{3}$

44. $\cot x = 1$

45. $\sec x = -2$

46. $\sec x = 2$

47. $\csc x = \sqrt{2}$

48. $\csc x = -\dfrac{2\sqrt{3}}{3}$

In Exercises 49 and 50, use the graph of the function to determine whether the function is even, odd, or neither.

49. $f(x) = \sec x$

50. $f(x) = \tan x$

51. ***Graphical Reasoning*** Consider the functions given by

$$f(x) = 2 \sin x \quad \text{and} \quad g(x) = \frac{1}{2} \csc x$$

on the interval $(0, \pi)$.

(a) Graph f and g in the same coordinate plane.

(b) Approximate the interval in which $f > g$.

(c) Describe the behavior of each of the functions as x approaches π. How is the behavior of g related to the behavior of f as x approaches π?

52. ***Graphical Reasoning*** Consider the functions given by

$$f(x) = \tan \frac{\pi x}{2} \quad \text{and} \quad g(x) = \frac{1}{2} \sec \frac{\pi x}{2}$$

on the interval $(-1, 1)$.

(a) Use a graphing utility to graph f and g in the same viewing window.

(b) Approximate the interval in which $f < g$.

(c) Approximate the interval in which $2f < 2g$. How does the result compare with that of part (b)? Explain.

In Exercises 53–56, use a graphing utility to graph the two equations in the same viewing window. Use the graphs to determine whether the expressions are equivalent. Verify the results algebraically.

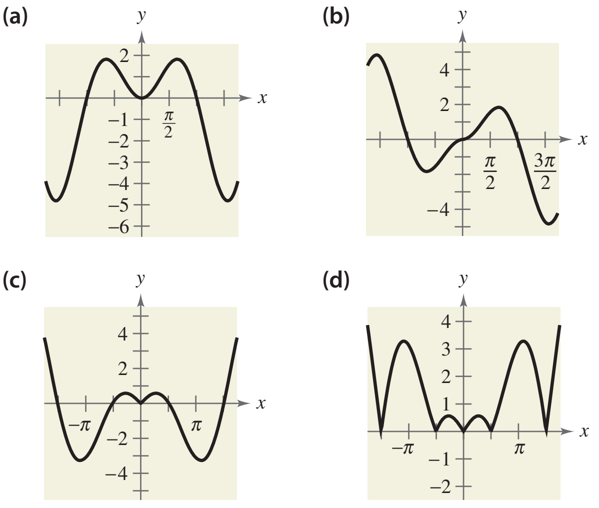
53. $y_1 = \sin x \csc x, \quad y_2 = 1$

54. $y_1 = \sin x \sec x, \quad y_2 = \tan x$

55. $y_1 = \dfrac{\cos x}{\sin x}, \quad y_2 = \cot x$

56. $y_1 = \sec^2 x - 1, \quad y_2 = \tan^2 x$

In Exercises 57–60, match the function with its graph. Describe the behavior of the function as x approaches zero. [The graphs are labeled (a), (b), (c), and (d).]



57. $f(x) = |x \cos x|$

58. $f(x) = x \sin x$

59. $g(x) = |x| \sin x$

60. $g(x) = |x| \cos x$

Conjecture **In Exercises 61–64, graph the functions f and g. Use the graphs to make a conjecture about the relationship between the functions.**

61. $f(x) = \sin x + \cos\left(x + \dfrac{\pi}{2}\right), \quad g(x) = 0$

62. $f(x) = \sin x - \cos\left(x + \dfrac{\pi}{2}\right), \quad g(x) = 2 \sin x$

63. $f(x) = \sin^2 x, \quad g(x) = \frac{1}{2}(1 - \cos 2x)$

64. $f(x) = \cos^2 \dfrac{\pi x}{2}, \quad g(x) = \dfrac{1}{2}(1 + \cos \pi x)$

In Exercises 65–68, use a graphing utility to graph the function and the damping factor of the function in the same viewing window. Describe the behavior of the function as x increases without bound.

65. $g(x) = e^{-x^2/2} \sin x$

66. $f(x) = e^{-x} \cos x$

67. $f(x) = 2^{-x/4} \cos \pi x$

68. $h(x) = 2^{-x^2/4} \sin x$

Exploration **In Exercises 69–74, use a graphing utility to graph the function. Describe the behavior of the function as x approaches zero.**

69. $y = \dfrac{6}{x} + \cos x, \quad x > 0$

70. $y = \dfrac{4}{x} + \sin 2x, \quad x > 0$

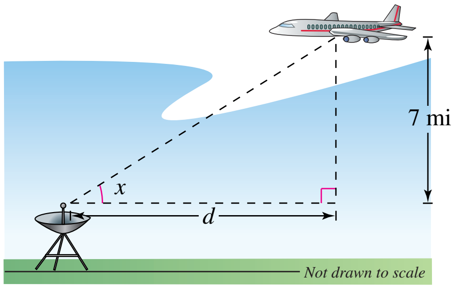
71. $g(x) = \dfrac{\sin x}{x}$

72. $f(x) = \dfrac{1 - \cos x}{x}$

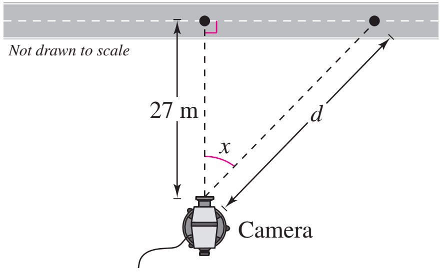
73. $f(x) = \sin \dfrac{1}{x}$

74. $h(x) = x \sin \dfrac{1}{x}$

75. ***Distance*** A plane flying at an altitude of 7 miles above a radar antenna will pass directly over the radar antenna (see figure). Let d be the ground distance from the antenna to the point directly under the plane and let x be the angle of elevation to the plane from the antenna. (d is positive as the plane approaches the antenna.) Write d as a function of x and graph the function over the interval $0 < x < \pi$.



76. ***Television Coverage*** A television camera is on a reviewing platform 27 meters from the street on which a parade will be passing from left to right (see figure). Write the distance d from the camera to a particular unit in the parade as a function of the angle x, and graph the function over the interval $-\pi/2 < x < \pi/2$. (Consider x as negative when a unit in the parade approaches from the left.)



Model It

77. ***Predator-Prey Model*** The population C of coyotes (a predator) at time t (in months) in a region is estimated to be

$$C = 5000 + 2000 \sin \frac{\pi t}{12}$$

and the population R of rabbits (its prey) is estimated to be

Model It (continued)

$$R = 25{,}000 + 15{,}000 \cos \frac{\pi t}{12}.$$

(a) Use a graphing utility to graph both models in the same viewing window. Use the window setting $0 \le t \le 100$.

(b) Use the graphs of the models in part (a) to explain the oscillations in the size of each population.

(c) The cycles of each population follow a periodic pattern. Find the period of each model and describe several factors that could be contributing to the cyclical patterns.

78. ***Sales*** The projected monthly sales S (in thousands of units) of lawn mowers (a seasonal product) are modeled by $S = 74 + 3t - 40 \cos(\pi t/6)$, where t is the time (in months), with $t = 1$ corresponding to January. Graph the sales function over 1 year.

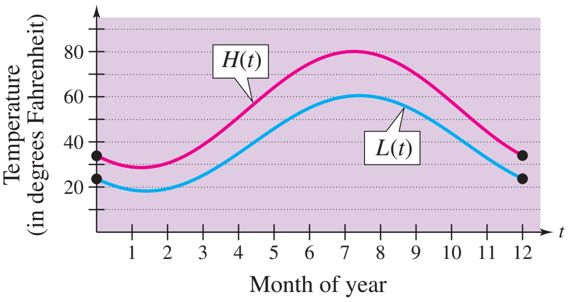
79. ***Meterology*** The normal monthly high temperatures H (in degrees Fahrenheit) for Erie, Pennsylvania are approximated by

$$H(t) = 54.33 - 20.38 \cos \frac{\pi t}{6} - 15.69 \sin \frac{\pi t}{6}$$

and the normal monthly low temperatures L are approximated by

$$L(t) = 39.36 - 15.70 \cos \frac{\pi t}{6} - 14.16 \sin \frac{\pi t}{6}$$

where t is the time (in months), with $t = 1$ corresponding to January (see figure). (Source: National Oceanic and Atmospheric Administration)



(a) What is the period of each function?

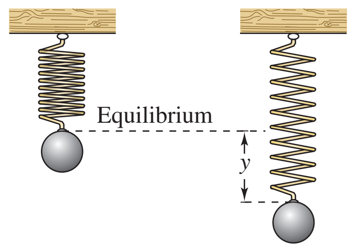
(b) During what part of the year is the difference between the normal high and normal low temperatures greatest? When is it smallest?

(c) The sun is northernmost in the sky around June 21, but the graph shows the warmest temperatures at a later date. Approximate the lag time of the temperatures relative to the position of the sun.

80. ***Harmonic Motion*** An object weighing W pounds is suspended from the ceiling by a steel spring (see figure). The weight is pulled downward (positive direction) from its equilibrium position and released. The resulting motion of the weight is described by the function

$$y = \frac{1}{2}e^{-t/4}\cos 4t, \quad t > 0$$

where y is the distance (in feet) and t is the time (in seconds).



(a) Use a graphing utility to graph the function.

(b) Describe the behavior of the displacement function for increasing values of time t.

Synthesis

True or False? **In Exercises 81 and 82, determine whether the statement is true or false. Justify your answer.**

81. The graph of $y = \csc x$ can be obtained on a calculator by graphing the reciprocal of $y = \sin x$.

82. The graph of $y = \sec x$ can be obtained on a calculator by graphing a translation of the reciprocal of $y = \sin x$.

83. ***Writing*** Describe the behavior of $f(x) = \tan x$ as x approaches $\pi/2$ from the left and from the right.

84. ***Writing*** Describe the behavior of $f(x) = \csc x$ as x approaches π from the left and from the right.

85. ***Exploration*** Consider the function given by

$$f(x) = x - \cos x.$$

(a) Use a graphing utility to graph the function and verify that there exists a zero between 0 and 1. Use the graph to approximate the zero.

(b) Starting with $x_0 = 1$, generate a sequence $x_1, x_2, x_3, \ldots$, where $x_n = \cos(x_{n-1})$. For example,

$$x_0 = 1$$
$$x_1 = \cos(x_0)$$
$$x_2 = \cos(x_1)$$
$$x_3 = \cos(x_2)$$
$$\vdots$$

What value does the sequence approach?

86. ***Approximation*** Using calculus, it can be shown that the tangent function can be approximated by the polynomial

$$\tan x \approx x + \frac{2x^3}{3!} + \frac{16x^5}{5!}$$

where x is in radians. Use a graphing utility to graph the tangent function and its polynomial approximation in the same viewing window. How do the graphs compare?

87. ***Approximation*** Using calculus, it can be shown that the secant function can be approximated by the polynomial

$$\sec x \approx 1 + \frac{x^2}{2!} + \frac{5x^4}{4!}$$

where x is in radians. Use a graphing utility to graph the secant function and its polynomial approximation in the same viewing window. How do the graphs compare?

88. ***Pattern Recognition***

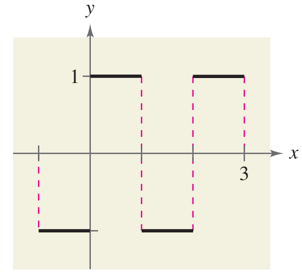
(a) Use a graphing utility to graph each function.

$$y_1 = \frac{4}{\pi}\left(\sin \pi x + \frac{1}{3}\sin 3\pi x\right)$$

$$y_2 = \frac{4}{\pi}\left(\sin \pi x + \frac{1}{3}\sin 3\pi x + \frac{1}{5}\sin 5\pi x\right)$$

(b) Identify the pattern started in part (a) and find a function y_3 that continues the pattern one more term. Use a graphing utility to graph y_3.

(c) The graphs in parts (a) and (b) approximate the periodic function in the figure. Find a function y_4 that is a better approximation.



Skills Review

In Exercises 89–92, solve the exponential equation. Round your answer to three decimal places.

89. $e^{2x} = 54$

90. $8^{3x} = 98$

91. $\dfrac{300}{1 + e^{-x}} = 100$

92. $\left(1 + \dfrac{0.15}{365}\right)^{365t} = 5$

In Exercises 93–98, solve the logarithmic equation. Round your answer to three decimal places.

93. $\ln(3x - 2) = 73$

94. $\ln(14 - 2x) = 68$

95. $\ln(x^2 + 1) = 3.2$

96. $\ln \sqrt{x + 4} = 5$

97. $\log_8 x + \log_8(x - 1) = \frac{1}{3}$

98. $\log_6 x + \log_6(x^2 - 1) = \log_6 64x$