

4.4 Trigonometric Functions of Any Angle

What you should learn

- Evaluate trigonometric functions of any angle.
- Use reference angles to evaluate trigonometric functions.
- Evaluate trigonometric functions of real numbers.

Why you should learn it

You can use trigonometric functions to model and solve real-life problems. For instance, in Exercise 87 on page 319, you can use trigonometric functions to model the monthly normal temperatures in New York City and Fairbanks, Alaska.



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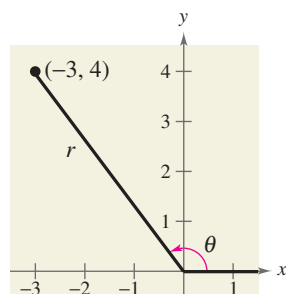


FIGURE 4.36

Introduction

In Section 4.3, the definitions of trigonometric functions were restricted to acute angles. In this section, the definitions are extended to cover *any* angle. If θ is an *acute* angle, these definitions coincide with those given in the preceding section.

Definitions of Trigonometric Functions of Any Angle

Let θ be an angle in standard position with (x, y) a point on the terminal side of θ and $r = \sqrt{x^2 + y^2} \neq 0$.

$$\sin \theta = \frac{y}{r}$$

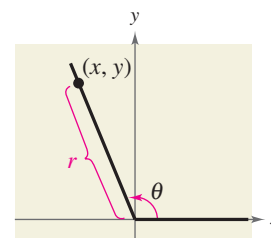
$$\cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x}, \quad x \neq 0$$

$$\cot \theta = \frac{x}{y}, \quad y \neq 0$$

$$\sec \theta = \frac{r}{x}, \quad x \neq 0$$

$$\csc \theta = \frac{r}{y}, \quad y \neq 0$$



Because $r = \sqrt{x^2 + y^2}$ *cannot* be zero, it follows that the sine and cosine functions are defined for any real value of θ . However, if $x = 0$, the tangent and secant of θ are undefined. For example, the tangent of 90° is undefined. Similarly, if $y = 0$, the cotangent and cosecant of θ are undefined.

Example 1 Evaluating Trigonometric Functions

Let $(-3, 4)$ be a point on the terminal side of θ . Find the sine, cosine, and tangent of θ .

Solution

Referring to Figure 4.36, you can see that $x = -3$, $y = 4$, and

$$r = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + 4^2} = \sqrt{25} = 5.$$

So, you have the following.

$$\sin \theta = \frac{y}{r} = \frac{4}{5}$$

$$\cos \theta = \frac{x}{r} = -\frac{3}{5}$$

$$\tan \theta = \frac{y}{x} = -\frac{4}{3}$$

CHECKPOINT Now try Exercise 1.

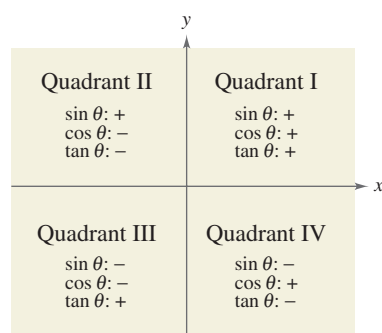
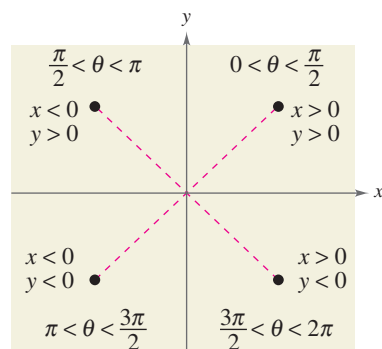


FIGURE 4.37

The *signs* of the trigonometric functions in the four quadrants can be determined easily from the definitions of the functions. For instance, because $\cos \theta = x/r$, it follows that $\cos \theta$ is positive wherever $x > 0$, which is in Quadrants I and IV. (Remember, r is always positive.) In a similar manner, you can verify the results shown in Figure 4.37.

Example 2 Evaluating Trigonometric Functions

Given $\tan \theta = -\frac{5}{4}$ and $\cos \theta > 0$, find $\sin \theta$ and $\sec \theta$.

Solution

Note that θ lies in Quadrant IV because that is the only quadrant in which the tangent is negative and the cosine is positive. Moreover, using

$$\tan \theta = \frac{y}{x} = -\frac{5}{4}$$

and the fact that y is negative in Quadrant IV, you can let $y = -5$ and $x = 4$. So, $r = \sqrt{16 + 25} = \sqrt{41}$ and you have

$$\begin{aligned}\sin \theta &= \frac{y}{r} = \frac{-5}{\sqrt{41}} \\ &\approx -0.7809\end{aligned}$$

$$\begin{aligned}\sec \theta &= \frac{r}{x} = \frac{\sqrt{41}}{4} \\ &\approx 1.6008.\end{aligned}$$



CHECKPOINT

Now try Exercise 17.

Example 3 Trigonometric Functions of Quadrant Angles

Evaluate the cosine and tangent functions at the four quadrant angles 0 , $\frac{\pi}{2}$, π , and $\frac{3\pi}{2}$.

Solution

To begin, choose a point on the terminal side of each angle, as shown in Figure 4.38. For each of the four points, $r = 1$, and you have the following.

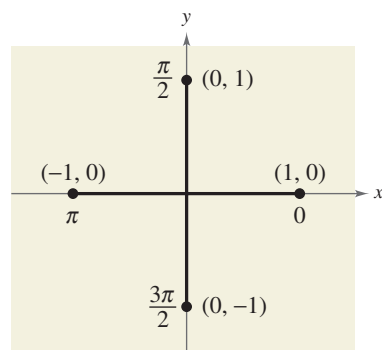


FIGURE 4.38

$$\cos 0 = \frac{x}{r} = \frac{1}{1} = 1 \quad \tan 0 = \frac{y}{x} = \frac{0}{1} = 0 \quad (x, y) = (1, 0)$$

$$\cos \frac{\pi}{2} = \frac{x}{r} = \frac{0}{1} = 0 \quad \tan \frac{\pi}{2} = \frac{y}{x} = \frac{1}{0} \Rightarrow \text{undefined} \quad (x, y) = (0, 1)$$

$$\cos \pi = \frac{x}{r} = \frac{-1}{1} = -1 \quad \tan \pi = \frac{y}{x} = \frac{0}{-1} = 0 \quad (x, y) = (-1, 0)$$

$$\cos \frac{3\pi}{2} = \frac{x}{r} = \frac{0}{1} = 0 \quad \tan \frac{3\pi}{2} = \frac{y}{x} = \frac{-1}{0} \Rightarrow \text{undefined} \quad (x, y) = (0, -1)$$



CHECKPOINT

Now try Exercise 29.

Sketching several angles with their reference angles may help reinforce the fact that the reference angle is the acute angle formed with the horizontal.

Reference Angles

The values of the trigonometric functions of angles greater than 90° (or less than 0°) can be determined from their values at corresponding acute angles called **reference angles**.

Definition of Reference Angle

Let θ be an angle in standard position. Its **reference angle** is the acute angle θ' formed by the terminal side of θ and the horizontal axis.

Figure 4.39 shows the reference angles for θ in Quadrants II, III, and IV.

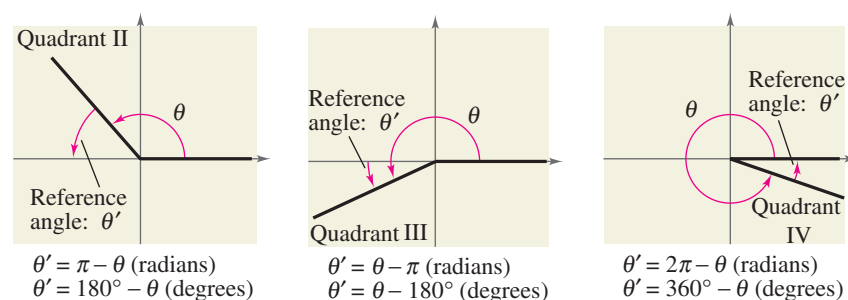


FIGURE 4.39

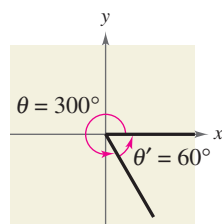


FIGURE 4.40

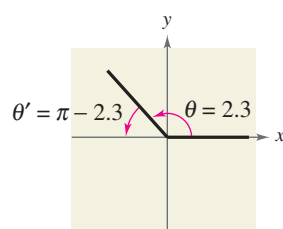


FIGURE 4.41

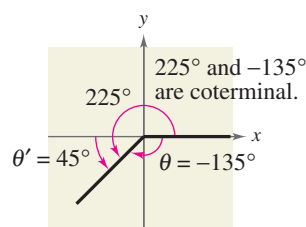


FIGURE 4.42

Example 4 Finding Reference Angles

Find the reference angle θ' .

- a. $\theta = 300^\circ$ b. $\theta = 2.3$ c. $\theta = -135^\circ$

Solution

- a. Because 300° lies in Quadrant IV, the angle it makes with the x -axis is

$$\begin{aligned}\theta' &= 360^\circ - 300^\circ \\ &= 60^\circ. \quad \text{Degrees}\end{aligned}$$

Figure 4.40 shows the angle $\theta = 300^\circ$ and its reference angle $\theta' = 60^\circ$.

- b. Because 2.3 lies between $\pi/2 \approx 1.5708$ and $\pi \approx 3.1416$, it follows that it is in Quadrant II and its reference angle is

$$\begin{aligned}\theta' &= \pi - 2.3 \\ &\approx 0.8416. \quad \text{Radians}\end{aligned}$$

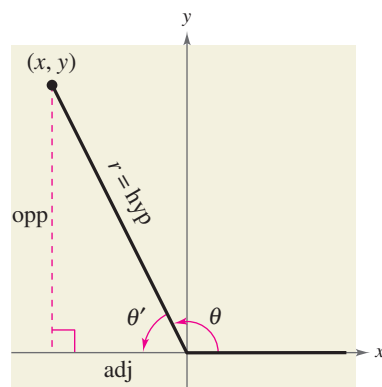
Figure 4.41 shows the angle $\theta = 2.3$ and its reference angle $\theta' = \pi - 2.3$.

- c. First, determine that -135° is coterminal with 225° , which lies in Quadrant III. So, the reference angle is

$$\begin{aligned}\theta' &= 225^\circ - 180^\circ \\ &= 45^\circ. \quad \text{Degrees}\end{aligned}$$

Figure 4.42 shows the angle $\theta = -135^\circ$ and its reference angle $\theta' = 45^\circ$.

CHECKPOINT Now try Exercise 37.



opp = $|y|$, adj = $|x|$

FIGURE 4.43

Trigonometric Functions of Real Numbers

To see how a reference angle is used to evaluate a trigonometric function, consider the point (x, y) on the terminal side of θ , as shown in Figure 4.43. By definition, you know that

$$\sin \theta = \frac{y}{r} \quad \text{and} \quad \tan \theta = \frac{y}{x}.$$

For the right triangle with acute angle θ' and sides of lengths $|x|$ and $|y|$, you have

$$\sin \theta' = \frac{\text{opp}}{\text{hyp}} = \frac{|y|}{r}$$

and

$$\tan \theta' = \frac{\text{opp}}{\text{adj}} = \frac{|y|}{|x|}.$$

So, it follows that $\sin \theta$ and $\sin \theta'$ are equal, *except possibly in sign*. The same is true for $\tan \theta$ and $\tan \theta'$ and for the other four trigonometric functions. In all cases, the sign of the function value can be determined by the quadrant in which θ lies.

Evaluating Trigonometric Functions of Any Angle

To find the value of a trigonometric function of any angle θ :

1. Determine the function value for the associated reference angle θ' .
2. Depending on the quadrant in which θ lies, affix the appropriate sign to the function value.

STUDY TIP

Learning the table of values at the right is worth the effort because doing so will increase both your efficiency and your confidence. Here is a pattern for the sine function that may help you remember the values.

θ	0°	30°	45°	60°	90°
$\sin \theta$	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$

Reverse the order to get cosine values of the same angles.

By using reference angles and the special angles discussed in the preceding section, you can greatly extend the scope of *exact* trigonometric values. For instance, knowing the function values of 30° means that you know the function values of all angles for which 30° is a reference angle. For convenience, the table below shows the exact values of the trigonometric functions of special angles and quadrant angles.

Trigonometric Values of Common Angles

θ (degrees)	0°	30°	45°	60°	90°	180°	270°
θ (radians)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$\sin \theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
$\tan \theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Undef.	0	Undef.

Example 5 Using Reference Angles

Evaluate each trigonometric function.

a. $\cos \frac{4\pi}{3}$ b. $\tan(-210^\circ)$ c. $\csc \frac{11\pi}{4}$

Solution

- a. Because $\theta = 4\pi/3$ lies in Quadrant III, the reference angle is $\theta' = (4\pi/3) - \pi = \pi/3$, as shown in Figure 4.44. Moreover, the cosine is negative in Quadrant III, so

$$\begin{aligned}\cos \frac{4\pi}{3} &= (-) \cos \frac{\pi}{3} \\ &= -\frac{1}{2}.\end{aligned}$$

Emphasize the importance of reference angles in evaluating trigonometric functions of angles greater than 90° .

- b. Because $-210^\circ + 360^\circ = 150^\circ$, it follows that -210° is coterminal with the second-quadrant angle 150° . So, the reference angle is $\theta' = 180^\circ - 150^\circ = 30^\circ$, as shown in Figure 4.45. Finally, because the tangent is negative in Quadrant II, you have

$$\begin{aligned}\tan(-210^\circ) &= (-) \tan 30^\circ \\ &= -\frac{\sqrt{3}}{3}.\end{aligned}$$

- c. Because $(11\pi/4) - 2\pi = 3\pi/4$, it follows that $11\pi/4$ is coterminal with the second-quadrant angle $3\pi/4$. So, the reference angle is $\theta' = \pi - (3\pi/4) = \pi/4$, as shown in Figure 4.46. Because the cosecant is positive in Quadrant II, you have

$$\begin{aligned}\csc \frac{11\pi}{4} &= (+) \csc \frac{\pi}{4} \\ &= \frac{1}{\sin(\pi/4)} \\ &= \sqrt{2}.\end{aligned}$$

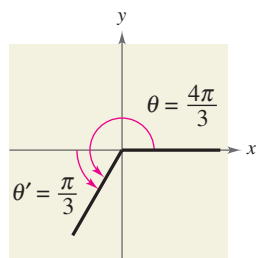


FIGURE 4.44

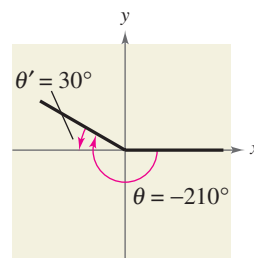


FIGURE 4.45

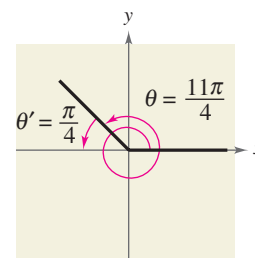


FIGURE 4.46

**CHECKPOINT**

Now try Exercise 51.

Example 6 Using Trigonometric Identities

Let θ be an angle in Quadrant II such that $\sin \theta = \frac{1}{3}$. Find (a) $\cos \theta$ and (b) $\tan \theta$ by using trigonometric identities.

Solution

a. Using the Pythagorean identity $\sin^2 \theta + \cos^2 \theta = 1$, you obtain

$$\left(\frac{1}{3}\right)^2 + \cos^2 \theta = 1 \quad \text{Substitute } \frac{1}{3} \text{ for } \sin \theta.$$

$$\cos^2 \theta = 1 - \frac{1}{9} = \frac{8}{9}.$$

Because $\cos \theta < 0$ in Quadrant II, you can use the negative root to obtain

$$\begin{aligned} \cos \theta &= -\frac{\sqrt{8}}{\sqrt{9}} \\ &= -\frac{2\sqrt{2}}{3}. \end{aligned}$$

b. Using the trigonometric identity $\tan \theta = \frac{\sin \theta}{\cos \theta}$, you obtain

$$\begin{aligned} \tan \theta &= \frac{1/3}{-2\sqrt{2}/3} \quad \text{Substitute for } \sin \theta \text{ and } \cos \theta. \\ &= -\frac{1}{2\sqrt{2}} \\ &= -\frac{\sqrt{2}}{4}. \end{aligned}$$

Students often have difficulty determining angles, especially when the functions given are csc, sec, and/or cot. Have your students rewrite the expression in terms of sin, cos, or tan, whichever is applicable, before evaluating.

Activities

1. Determine the exact values of the six trigonometric functions of the angle in standard position whose terminal side contains the point $(-3, -7)$.

Answer:

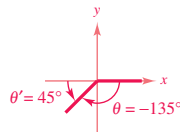
$$\sin \theta = -\frac{7}{\sqrt{58}} \quad \csc \theta = -\frac{\sqrt{58}}{7}$$

$$\cos \theta = -\frac{3}{\sqrt{58}} \quad \sec \theta = -\frac{\sqrt{58}}{3}$$

$$\tan \theta = \frac{7}{3} \quad \cot \theta = \frac{3}{7}$$

2. For the angle $\theta = -135^\circ$, find the reference angle θ' , and sketch θ and θ' in standard position.

Answer: $\theta' = 45^\circ$



3. Find two values of θ , $0 \leq \theta < 2\pi$, that satisfy the equation $\tan \theta = -1$. Do not use your calculator.

Answer: $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

CHECKPOINT Now try Exercise 59.

You can use a calculator to evaluate trigonometric functions, as shown in the next example.

Example 7 Using a Calculator

Use a calculator to evaluate each trigonometric function.

a. $\cot 410^\circ$ b. $\sin(-7)$ c. $\sec \frac{\pi}{9}$

Solution

Function	Mode	Calculator Keystrokes	Display
a. $\cot 410^\circ$	Degree	$\boxed{1} \boxed{\text{TAN}} \boxed{(\boxed{410})} \boxed{)} \boxed{x^{-1}} \boxed{\text{ENTER}}$	0.8390996
b. $\sin(-7)$	Radian	$\boxed{\text{SIN}} \boxed{(\boxed{-7})} \boxed{)} \boxed{\text{ENTER}}$	-0.6569866
c. $\sec \frac{\pi}{9}$	Radian	$\boxed{1} \boxed{\text{COS}} \boxed{(\boxed{\pi} \boxed{\div} \boxed{9})} \boxed{)} \boxed{x^{-1}} \boxed{\text{ENTER}}$	1.0641778

CHECKPOINT Now try Exercise 69.

4.4 Exercises

VOCABULARY CHECK:

In Exercises 1–6, let θ be an angle in standard position, with (x, y) a point on the terminal side of θ and $r = \sqrt{x^2 + y^2} \neq 0$.

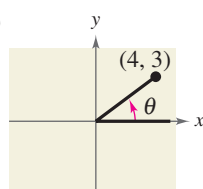
1. $\sin \theta =$ _____
2. $\frac{r}{y} =$ _____
3. $\tan \theta =$ _____
4. $\sec \theta =$ _____
5. $\frac{x}{r} =$ _____
6. $\frac{x}{y} =$ _____

7. The acute positive angle that is formed by the terminal side of the angle θ and the horizontal axis is called the _____ angle of θ and is denoted by θ' .

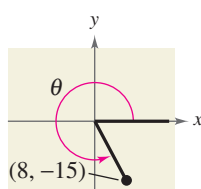
PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at www.Eduspace.com.

In Exercises 1–4, determine the exact values of the six trigonometric functions of the angle θ .

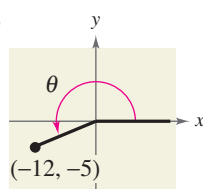
1. (a)



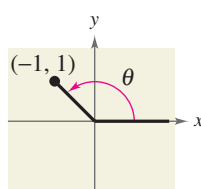
(b)



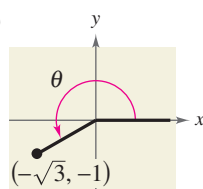
2. (a)



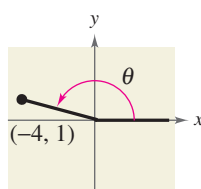
(b)



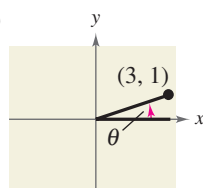
3. (a)



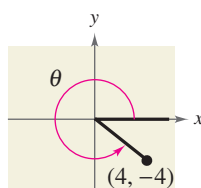
(b)



4. (a)



(b)



In Exercises 5–10, the point is on the terminal side of an angle in standard position. Determine the exact values of the six trigonometric functions of the angle.

5. $(7, 24)$
6. $(8, 15)$
7. $(-4, 10)$
8. $(-5, -2)$

9. $(-3.5, 6.8)$

10. $(3\frac{1}{2}, -7\frac{3}{4})$

In Exercises 11–14, state the quadrant in which θ lies.

11. $\sin \theta < 0$ and $\cos \theta < 0$
12. $\sin \theta > 0$ and $\cos \theta > 0$
13. $\sin \theta > 0$ and $\tan \theta < 0$
14. $\sec \theta > 0$ and $\cot \theta < 0$

In Exercises 15–24, find the values of the six trigonometric functions of θ with the given constraint.

Function Value	Constraint
15. $\sin \theta = \frac{3}{5}$	θ lies in Quadrant II.
16. $\cos \theta = -\frac{4}{5}$	θ lies in Quadrant III.
17. $\tan \theta = -\frac{15}{8}$	$\sin \theta < 0$
18. $\cos \theta = \frac{8}{17}$	$\tan \theta < 0$
19. $\cot \theta = -3$	$\cos \theta > 0$
20. $\csc \theta = 4$	$\cot \theta < 0$
21. $\sec \theta = -2$	$\sin \theta > 0$
22. $\sin \theta = 0$	$\sec \theta = -1$
23. $\cot \theta$ is undefined.	$\pi/2 \leq \theta \leq 3\pi/2$
24. $\tan \theta$ is undefined.	$\pi \leq \theta \leq 2\pi$

In Exercises 25–28, the terminal side of θ lies on the given line in the specified quadrant. Find the values of the six trigonometric functions of θ by finding a point on the line.

Line	Quadrant
25. $y = -x$	II
26. $y = \frac{1}{3}x$	III
27. $2x - y = 0$	III
28. $4x + 3y = 0$	IV

In Exercises 29–36, evaluate the trigonometric function of the quadrant angle.

29. $\sin \pi$
30. $\csc \frac{3\pi}{2}$
31. $\sec \frac{3\pi}{2}$
32. $\sec \pi$
33. $\sin \frac{\pi}{2}$
34. $\cot \pi$
35. $\csc \pi$
36. $\cot \frac{\pi}{2}$

In Exercises 37–44, find the reference angle θ' , and sketch θ and θ' in standard position.

37. $\theta = 203^\circ$
38. $\theta = 309^\circ$
39. $\theta = -245^\circ$
40. $\theta = -145^\circ$
41. $\theta = \frac{2\pi}{3}$
42. $\theta = \frac{7\pi}{4}$
43. $\theta = 3.5$
44. $\theta = \frac{11\pi}{3}$

In Exercises 45–58, evaluate the sine, cosine, and tangent of the angle without using a calculator.

45. 225°
46. 300°
47. 750°
48. -405°
49. -150°
50. -840°
51. $\frac{4\pi}{3}$
52. $\frac{\pi}{4}$
53. $-\frac{\pi}{6}$
54. $-\frac{\pi}{2}$
55. $\frac{11\pi}{4}$
56. $\frac{10\pi}{3}$
57. $-\frac{3\pi}{2}$
58. $-\frac{25\pi}{4}$

In Exercises 59–64, find the indicated trigonometric value in the specified quadrant.

Function	Quadrant	Trigonometric Value
59. $\sin \theta = -\frac{3}{5}$	IV	$\cos \theta$
60. $\cot \theta = -3$	II	$\sin \theta$
61. $\tan \theta = \frac{3}{2}$	III	$\sec \theta$
62. $\csc \theta = -2$	IV	$\cot \theta$
63. $\cos \theta = \frac{5}{8}$	I	$\sec \theta$
64. $\sec \theta = -\frac{9}{4}$	III	$\tan \theta$

In Exercises 65–80, use a calculator to evaluate the trigonometric function. Round your answer to four decimal places. (Be sure the calculator is set in the correct angle mode.)


65. $\sin 10^\circ$
66. $\sec 225^\circ$
67. $\cos(-110^\circ)$
68. $\csc(-330^\circ)$
69. $\tan 304^\circ$
70. $\cot 178^\circ$
71. $\sec 72^\circ$
72. $\tan(-188^\circ)$
73. $\tan 4.5$
74. $\cot 1.35$
75. $\tan \frac{\pi}{9}$
76. $\tan\left(-\frac{\pi}{9}\right)$
77. $\sin(-0.65)$
78. $\sec 0.29$
79. $\cot\left(-\frac{11\pi}{8}\right)$
80. $\csc\left(-\frac{15\pi}{14}\right)$

In Exercises 81–86, find two solutions of the equation. Give your answers in degrees ($0^\circ \leq \theta < 360^\circ$) and in radians ($0 \leq \theta < 2\pi$). Do not use a calculator.

81. (a) $\sin \theta = \frac{1}{2}$
- (b) $\sin \theta = -\frac{1}{2}$
82. (a) $\cos \theta = \frac{\sqrt{2}}{2}$
- (b) $\cos \theta = -\frac{\sqrt{2}}{2}$
83. (a) $\csc \theta = \frac{2\sqrt{3}}{3}$
- (b) $\cot \theta = -1$
84. (a) $\sec \theta = 2$
- (b) $\sec \theta = -2$
85. (a) $\tan \theta = 1$
- (b) $\cot \theta = -\sqrt{3}$
86. (a) $\sin \theta = \frac{\sqrt{3}}{2}$
- (b) $\sin \theta = -\frac{\sqrt{3}}{2}$

Model It

87. Data Analysis: Meteorology The table shows the monthly normal temperatures (in degrees Fahrenheit) for selected months for New York City (N) and Fairbanks, Alaska (F). (Source: National Climatic Data Center)



Month	New York City, N	Fairbanks, F
January	33	-10
April	52	32
July	77	62
October	58	24
December	38	-6

- (a) Use the *regression* feature of a graphing utility to find a model of the form
- $$y = a \sin(bt + c) + d$$
- for each city. Let t represent the month, with $t = 1$ corresponding to January.

Model It (continued)

- (b) Use the models from part (a) to find the monthly normal temperatures for the two cities in February, March, May, June, August, September, and November.
- (c) Compare the models for the two cities.

- 88. Sales** A company that produces snowboards, which are seasonal products, forecasts monthly sales over the next 2 years to be

$$S = 23.1 + 0.442t + 4.3 \cos \frac{\pi t}{6}$$

where S is measured in thousands of units and t is the time in months, with $t = 1$ representing January 2006. Predict sales for each of the following months.

- (a) February 2006
 (b) February 2007
 (c) June 2006
 (d) June 2007

- 89. Harmonic Motion** The displacement from equilibrium of an oscillating weight suspended by a spring is given by

$$y(t) = 2 \cos 6t$$

where y is the displacement (in centimeters) and t is the time (in seconds). Find the displacement when (a) $t = 0$, (b) $t = \frac{1}{4}$, and (c) $t = \frac{1}{2}$.

- 90. Harmonic Motion** The displacement from equilibrium of an oscillating weight suspended by a spring and subject to the damping effect of friction is given by

$$y(t) = 2e^{-t} \cos 6t$$

where y is the displacement (in centimeters) and t is the time (in seconds). Find the displacement when (a) $t = 0$, (b) $t = \frac{1}{4}$, and (c) $t = \frac{1}{2}$.

- 91. Electric Circuits** The current I (in amperes) when 100 volts is applied to a circuit is given by

$$I = 5e^{-2t} \sin t$$

where t is the time (in seconds) after the voltage is applied. Approximate the current at $t = 0.7$ second after the voltage is applied.

- 92. Distance** An airplane, flying at an altitude of 6 miles, is on a flight path that passes directly over an observer (see figure). If θ is the angle of elevation from the observer to the plane, find the distance d from the observer to the plane when (a) $\theta = 30^\circ$, (b) $\theta = 90^\circ$, and (c) $\theta = 120^\circ$.

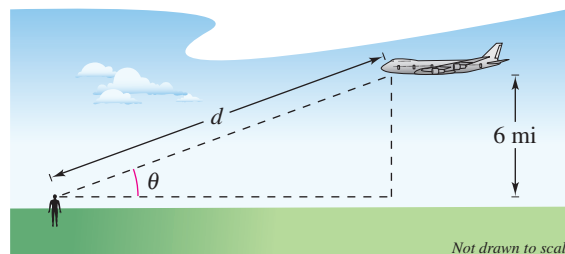
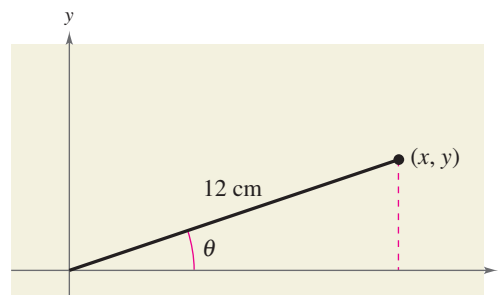


FIGURE FOR 92

Synthesis

True or False? In Exercises 93 and 94, determine whether the statement is true or false. Justify your answer.

- 93.** In each of the four quadrants, the signs of the secant function and sine function will be the same.
- 94.** To find the reference angle for an angle θ (given in degrees), find the integer n such that $0 \leq 360^\circ n - \theta \leq 360^\circ$. The difference $360^\circ n - \theta$ is the reference angle.
- 95. Writing** Consider an angle in standard position with $r = 12$ centimeters, as shown in the figure. Write a short paragraph describing the changes in the values of x , y , $\sin \theta$, $\cos \theta$, and $\tan \theta$ as θ increases continuously from 0° to 90° .



- 96. Writing** Explain how reference angles are used to find the trigonometric functions of obtuse angles.

Skills Review

In Exercises 97–106, graph the function. Identify the domain and any intercepts and asymptotes of the function.

- 97.** $y = x^2 + 3x - 4$ **98.** $y = 2x^2 - 5x$
99. $f(x) = x^3 + 8$ **100.** $g(x) = x^4 + 2x^2 - 3$
101. $f(x) = \frac{x-7}{x^2+4x+4}$ **102.** $h(x) = \frac{x^2-1}{x+5}$
103. $y = 2^{x-1}$ **104.** $y = 3^{x+1} + 2$
105. $y = \ln x^4$ **106.** $y = \log_{10}(x+2)$