What you should learn
• Solve polynomial inequalities.
• Solve rational inequalities.
• Use inequalities to model and solve real-life problems.

Why you should learn it
Inequalities can be used to model and solve real-life problems. For instance, in Exercise 73 on page 205, a polynomial inequality is used to model the percent of households that own a television and have cable in the United States.

Polynomial Inequalities
To solve a polynomial inequality such as \( x^2 - 2x - 3 < 0 \), you can use the fact that a polynomial can change signs only at its zeros (the \( x \)-values that make the polynomial equal to zero). Between two consecutive zeros, a polynomial must be entirely positive or entirely negative. This means that when the real zeros of a polynomial are put in order, they divide the real number line into intervals in which the polynomial has no sign changes. These zeros are the critical numbers of the inequality, and the resulting intervals are the test intervals for the inequality. For instance, the polynomial above factors as

\[
x^2 - 2x - 3 = (x + 1)(x - 3)
\]

and has two zeros, \( x = -1 \) and \( x = 3 \). These zeros divide the real number line into three test intervals:

\[
(-\infty, -1), (-1, 3), \text{ and } (3, \infty).
\]

(See Figure 2.51.)

So, to solve the inequality \( x^2 - 2x - 3 < 0 \), you need only test one value from each of these test intervals to determine whether the value satisfies the original inequality. If so, you can conclude that the interval is a solution of the inequality.

Finding Test Intervals for a Polynomial
To determine the intervals on which the values of a polynomial are entirely negative or entirely positive, use the following steps.

1. Find all real zeros of the polynomial, and arrange the zeros in increasing order (from smallest to largest). These zeros are the critical numbers of the polynomial.

2. Use the critical numbers of the polynomial to determine its test intervals.

3. Choose one representative \( x \)-value in each test interval and evaluate the polynomial at that value. If the value of the polynomial is negative, the polynomial will have negative values for every \( x \)-value in the interval. If the value of the polynomial is positive, the polynomial will have positive values for every \( x \)-value in the interval.
Chapter 2 Polynomial and Rational Functions

Example 1 Solving a Polynomial Inequality

Solve
\[ x^2 - x - 6 < 0. \]

Solution
By factoring the polynomial as
\[ x^2 - x - 6 = (x + 2)(x - 3), \]
you can see that the critical numbers are \( x = -2 \) and \( x = 3 \). So, the polynomial’s test intervals are
\( (-\infty, -2), \ ( -2, 3), \text{ and } (3, \infty). \)

In each test interval, choose a representative \( x \)-value and evaluate the polynomial.

<table>
<thead>
<tr>
<th>Test Interval</th>
<th>( x )-Value</th>
<th>Polynomial Value</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (-\infty, -2) )</td>
<td>( x = -3 )</td>
<td>( (-3)^2 - (-3) - 6 = 6 )</td>
<td>Positive</td>
</tr>
<tr>
<td>( (-2, 3) )</td>
<td>( x = 0 )</td>
<td>( (0)^2 - (0) - 6 = -6 )</td>
<td>Negative</td>
</tr>
<tr>
<td>( (3, \infty) )</td>
<td>( x = 4 )</td>
<td>( (4)^2 - (4) - 6 = 6 )</td>
<td>Positive</td>
</tr>
</tbody>
</table>

From this you can conclude that the inequality is satisfied for all \( x \)-values in \( (-2, 3) \). This implies that the solution of the inequality \( x^2 - x - 6 < 0 \) is the interval \( (-2, 3) \), as shown in Figure 2.52. Note that the original inequality contains a less than symbol. This means that the solution set does not contain the endpoints of the test interval \( (-2, 3) \).

Choose \( x = -3 \).
\[ (x + 2)(x - 3) > 0 \]

Choose \( x = 4 \).
\[ (x + 2)(x - 3) > 0 \]

Choose \( x = 0 \).
\[ (x + 2)(x - 3) < 0 \]

Figure 2.52

Now try Exercise 13.

As with linear inequalities, you can check the reasonableness of a solution by substituting \( x \)-values into the original inequality. For instance, to check the solution found in Example 1, try substituting several \( x \)-values from the interval \( (-2, 3) \) into the inequality
\[ x^2 - x - 6 < 0. \]

Regardless of which \( x \)-values you choose, the inequality should be satisfied.

You can also use a graph to check the result of Example 1. Sketch the graph of \( y = x^2 - x - 6 \), as shown in Figure 2.53. Notice that the graph is below the \( x \)-axis on the interval \( (-2, 3) \).

Remind students that critical numbers are numbers at which the polynomial can change signs and are not necessarily parts of the solutions of the inequality.
In Example 1, the polynomial inequality was given in general form (with the polynomial on one side and zero on the other). Whenever this is not the case, you should begin the solution process by writing the inequality in general form.

**Example 2  Solving a Polynomial Inequality**

Solve \(2x^3 - 3x^2 - 32x > -48\).

**Solution**

Begin by writing the inequality in general form.

\[
2x^3 - 3x^2 - 32x > -48
\]

Write original inequality.

\[
2x^3 - 3x^2 - 32x + 48 > 0
\]

Write in general form.

\[
(x - 4)(x + 4)(2x - 3) > 0
\]

Factor.

The critical numbers are \(x = -4, x = \frac{3}{2}, \) and \(x = 4,\) and the test intervals are \(( -\infty, -4), (-4, \frac{3}{2}), (\frac{3}{2}, 4),\) and \((4, \infty).\)

<table>
<thead>
<tr>
<th>Test Interval</th>
<th>(x)-Value</th>
<th>Polynomial Value</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-\infty, -4))</td>
<td>(x = -5)</td>
<td>(2(-5)^3 - 3(-5)^2 - 32(-5) + 48)</td>
<td>Negative</td>
</tr>
<tr>
<td>((-4, \frac{3}{2}))</td>
<td>(x = 0)</td>
<td>(2(0)^3 - 3(0)^2 - 32(0) + 48)</td>
<td>Positive</td>
</tr>
<tr>
<td>((\frac{3}{2}, 4))</td>
<td>(x = 2)</td>
<td>(2(2)^3 - 3(2)^2 - 32(2) + 48)</td>
<td>Negative</td>
</tr>
<tr>
<td>((4, \infty))</td>
<td>(x = 5)</td>
<td>(2(5)^3 - 3(5)^2 - 32(5) + 48)</td>
<td>Positive</td>
</tr>
</tbody>
</table>

From this you can conclude that the inequality is satisfied on the open intervals \((-4, \frac{3}{2})\) and \((4, \infty).\) Therefore, the solution set consists of all real numbers in the intervals \((-4, \frac{3}{2})\) and \((4, \infty),\) as shown in Figure 2.54.

**Figure 2.54**

\[
\text{Choose } x = 0, \quad (x - 4)(x + 4)(2x - 3) > 0
\]

\[
\text{Choose } x = 5, \quad (x - 4)(x + 4)(2x - 3) > 0
\]

\[
\text{Choose } x = -5, \quad (x - 4)(x + 4)(2x - 3) < 0
\]

\[
\text{Choose } x = 2, \quad (x - 4)(x + 4)(2x - 3) < 0
\]

When solving a polynomial inequality, be sure you have accounted for the particular type of inequality symbol given in the inequality. For instance, in Example 2, note that the original inequality contained a “greater than” symbol and the solution consisted of two open intervals. If the original inequality had been

\[
2x^3 - 3x^2 - 32x \geq -48
\]

the solution would have consisted of the closed interval \([-4, \frac{3}{2}]\) and the interval \([4, \infty).\)
Each of the polynomial inequalities in Examples 1 and 2 has a solution set that consists of a single interval or the union of two intervals. When solving the exercises for this section, watch for unusual solution sets, as illustrated in Example 3.

**Example 3  Unusual Solution Sets**

a. The solution set of the following inequality consists of the entire set of real numbers, \((-\infty, \infty)\). In other words, the value of the quadratic \(x^2 + 2x + 4\) is positive for every real value of \(x\).

\[x^2 + 2x + 4 > 0\]

b. The solution set of the following inequality consists of the single real number \((-1)\), because the quadratic \(x^2 + 2x + 1\) has only one critical number, \(x = -1\), and it is the only value that satisfies the inequality.

\[x^2 + 2x + 1 \leq 0\]

c. The solution set of the following inequality is empty. In other words, the quadratic \(x^2 + 3x + 5\) is not less than zero for any value of \(x\).

\[x^2 + 3x + 5 < 0\]

d. The solution set of the following inequality consists of all real numbers except \(x = 2\). In interval notation, this solution set can be written as \((-\infty, 2) \cup (2, \infty)\).

\[x^2 - 4x + 4 > 0\]

Activity
Solve the four inequalities in Example 3 using the method shown in Example 1.

Exploration
You can use a graphing utility to verify the results in Example 3. For instance, the graph of \(y = x^2 + 2x + 4\) is shown below. Notice that the y-values are greater than 0 for all values of \(x\), as stated in Example 3(a).

Use the graphing utility to graph the following:

\[y = x^2 + 2x + 1 \quad y = x^2 + 3x + 5 \quad y = x^2 - 4x + 4\]

Explain how you can use the graphs to verify the results of parts (b), (c), and (d) of Example 3.
Rational Inequalities

The concepts of critical numbers and test intervals can be extended to rational inequalities. To do this, use the fact that the value of a rational expression can change sign only at its zeros (the x-values for which its numerator is zero) and its undefined values (the x-values for which its denominator is zero). These two types of numbers make up the critical numbers of a rational inequality. When solving a rational inequality, begin by writing the inequality in general form with the rational expression on the left and zero on the right.

**Example 4**  Solving a Rational Inequality

Solve \( \frac{2x - 7}{x - 5} \leq 3. \)

**Solution**

\[
\frac{2x - 7}{x - 5} \leq 3 \quad \text{Write original inequality.}
\]

\[
\frac{2x - 7}{x - 5} - 3 \leq 0 \quad \text{Write in general form.}
\]

\[
\frac{2x - 7 - 3x + 15}{x - 5} \leq 0 \quad \text{Find the LCD and add fractions.}
\]

\[
\frac{-x + 8}{x - 5} \leq 0 \quad \text{Simplify.}
\]

Critical numbers: \( x = 5, x = 8 \)  Zeros and undefined values of rational expression

Test intervals: \( (-\infty, 5), (5, 8), (8, \infty) \)

Test: \( \frac{-x + 8}{x - 5} \leq 0? \)

After testing these intervals, as shown in Figure 2.55, you can see that the inequality is satisfied on the open intervals \( (-\infty, 5) \) and \( (8, \infty) \). Moreover, because \( \frac{-x + 8}{x - 5} = 0 \) when \( x = 8 \), you can conclude that the solution set consists of all real numbers in the intervals \( (-\infty, 5) \cup [8, \infty) \). (Be sure to use a closed interval to indicate that \( x \) can equal 8.)

For Exercise 39.
Applications

One common application of inequalities comes from business and involves profit, revenue, and cost. The formula that relates these three quantities is

\[
P = R - C.
\]

**Example 5** Increasing the Profit for a Product

The marketing department of a calculator manufacturer has determined that the demand for a new model of calculator is

\[
p = 100 - 0.00001x, \quad 0 \leq x \leq 10,000,000
\]

where \( p \) is the price per calculator (in dollars) and \( x \) represents the number of calculators sold. (If this model is accurate, no one would be willing to pay $100 for the calculator. At the other extreme, the company couldn’t sell more than 10 million calculators.) The revenue for selling \( x \) calculators is

\[
R = xp = x(100 - 0.00001x)
\]

as shown in Figure 2.56. The total cost of producing \( x \) calculators is $10 per calculator plus a development cost of $2,500,000. So, the total cost is

\[
C = 10x + 2,500,000.
\]

What price should the company charge per calculator to obtain a profit of at least $190,000,000?

**Solution**

Verbal Model:

\[
\text{Profit} = \text{Revenue} - \text{Cost}
\]

Equation:

\[
P = R - C
\]

\[
P = 100x - 0.00001x^2 - (10x + 2,500,000)
\]

\[
P = -0.00001x^2 + 90x - 2,500,000
\]

To answer the question, solve the inequality

\[
P \geq 190,000,000
\]

\[
-0.00001x^2 + 90x - 2,500,000 \geq 190,000,000.
\]

When you write the inequality in general form, find the critical numbers and the test intervals, and then test a value in each test interval, you can find the solution to be

\[
3,500,000 \leq x \leq 5,500,000
\]

as shown in Figure 2.57. Substituting the \( x \)-values in the original price equation shows that prices of

\[
$45.00 \leq p \leq $65.00
\]

will yield a profit of at least $190,000,000.

**Checkpoint** Now try Exercise 71.
Another common application of inequalities is finding the domain of an expression that involves a square root, as shown in Example 6.

**Example 6 Finding the Domain of an Expression**

Find the domain of $\sqrt{64 - 4x^2}$.

**Algebraic Solution**

Remember that the domain of an expression is the set of all $x$-values for which the expression is defined. Because $\sqrt{64 - 4x^2}$ is defined (has real values) only if $64 - 4x^2$ is nonnegative, the domain is given by $64 - 4x^2 \geq 0$.

\[
64 - 4x^2 \geq 0 \quad \text{Write in general form.}
\]
\[
16 - x^2 \geq 0 \quad \text{Divide each side by 4.}
\]
\[
(4 - x)(4 + x) \geq 0 \quad \text{Write in factored form.}
\]

So, the inequality has two critical numbers: $x = -4$ and $x = 4$. You can use these two numbers to test the inequality as follows.

**Critical numbers:** $x = -4, x = 4$

**Test intervals:** $(-\infty, -4), (-4, 4), (4, \infty)$

**Test:** For what values of $x$ is $\sqrt{64 - 4x^2} \geq 0$?

A test shows that the inequality is satisfied in the closed interval $[-4, 4]$. So, the domain of the expression $\sqrt{64 - 4x^2}$ is the interval $[-4, 4]$.

**Graphical Solution**

Begin by sketching the graph of the equation $y = \sqrt{64 - 4x^2}$, as shown in Figure 2.58. From the graph, you can determine that the $x$-values extend from $-4$ to $4$ (including $-4$ and $4$). So, the domain of the expression $\sqrt{64 - 4x^2}$ is the interval $[-4, 4]$.

To analyze a test interval, choose a representative $x$-value in the interval and evaluate the expression at that value. For instance, in Example 6, if you substitute any number from the interval $[-4, 4]$ into the expression $\sqrt{64 - 4x^2}$ you will obtain a nonnegative number under the radical symbol that simplifies to a real number. If you substitute any number from the intervals $(-\infty, -4)$ and $(4, \infty)$ you will obtain a complex number. It might be helpful to draw a visual representation of the intervals as shown in Figure 2.59.

**Writing about Mathematics**

**Profit Analysis**

Consider the relationship

\[ P = R - C \]

described on page 202. Write a paragraph discussing why it might be beneficial to solve $P < 0$ if you owned a business. Use the situation described in Example 5 to illustrate your reasoning.

**Additional Examples**

a. Find the domain of $\sqrt{2x^2 - 18}$.
   **Answer:** $(-\infty, -3) \cup [3, \infty)$

b. Find the domain of $\frac{2}{\sqrt{27 - 3x^2}}$.
   **Answer:** $(-3, 3)$
Exercise 2.7

VOCABULARY CHECK: Fill in the blanks.
1. To solve a polynomial inequality, find the _______ numbers of the polynomial, and use these numbers to create _______ _______ for the inequality.
2. The critical numbers of a rational expression are its _______ and its _______ _______.
3. The formula that relates cost, revenue, and profit is _______.


In Exercises 1–4, determine whether each value of \( x \) is a solution of the inequality.

<table>
<thead>
<tr>
<th>Inequality</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( x^2 - 3 &lt; 0 )</td>
<td>(a) ( x = 3 ) (b) ( x = 0 ) (c) ( x = \frac{3}{2} ) (d) ( x = -5 )</td>
</tr>
<tr>
<td>2. ( x^2 - x - 12 \geq 0 )</td>
<td>(a) ( x = 5 ) (b) ( x = 0 ) (c) ( x = -4 ) (d) ( x = -3 )</td>
</tr>
<tr>
<td>3. ( \frac{x + 2}{x - 4} \geq 3 )</td>
<td>(a) ( x = 5 ) (b) ( x = 4 ) (c) ( x = -\frac{2}{3} ) (d) ( x = \frac{2}{3} )</td>
</tr>
<tr>
<td>4. ( \frac{3x^2}{x^2 + 4} &lt; 1 )</td>
<td>(a) ( x = -2 ) (b) ( x = -1 ) (c) ( x = 0 ) (d) ( x = 3 )</td>
</tr>
</tbody>
</table>

In Exercises 5–8, find the critical numbers of the expression.

<table>
<thead>
<tr>
<th>Exercise</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.</td>
<td>( 2x^2 - x - 6 )</td>
</tr>
<tr>
<td>6.</td>
<td>( 9x^3 - 25x^2 )</td>
</tr>
<tr>
<td>7.</td>
<td>( 2 + \frac{3}{x - 5} )</td>
</tr>
<tr>
<td>8.</td>
<td>( \frac{x}{x + 2} - \frac{2}{x - 1} )</td>
</tr>
</tbody>
</table>

In Exercises 9–26, solve the inequality and graph the solution on the real number line.

<table>
<thead>
<tr>
<th>Exercise</th>
<th>Inequality</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.</td>
<td>( x^2 \leq 9 )</td>
</tr>
<tr>
<td>10.</td>
<td>( x^2 &lt; 36 )</td>
</tr>
<tr>
<td>11.</td>
<td>( (x + 2)^2 &lt; 25 )</td>
</tr>
<tr>
<td>12.</td>
<td>( (x - 3)^2 \geq 1 )</td>
</tr>
<tr>
<td>13.</td>
<td>( x^2 + 4x + 4 \geq 9 )</td>
</tr>
<tr>
<td>14.</td>
<td>( x^2 - 6x + 9 &lt; 16 )</td>
</tr>
<tr>
<td>15.</td>
<td>( x^2 + x &lt; 6 )</td>
</tr>
<tr>
<td>16.</td>
<td>( x^2 + 2x &gt; 3 )</td>
</tr>
<tr>
<td>17.</td>
<td>( x^2 + 2x - 3 &lt; 0 )</td>
</tr>
<tr>
<td>18.</td>
<td>( x^2 - 4x - 1 &gt; 0 )</td>
</tr>
<tr>
<td>19.</td>
<td>( x^2 + 8x - 5 \geq 0 )</td>
</tr>
<tr>
<td>20.</td>
<td>( -2x^2 + 6x + 15 \leq 0 )</td>
</tr>
<tr>
<td>21.</td>
<td>( x^2 - 3x^2 - x + 3 &gt; 0 )</td>
</tr>
<tr>
<td>22.</td>
<td>( x^2 + 2x^2 - 4x - 8 \geq 0 )</td>
</tr>
<tr>
<td>23.</td>
<td>( x^2 - 2x^2 - 9x - 2 \geq -20 )</td>
</tr>
<tr>
<td>24.</td>
<td>( 4x^3 + 13x^2 - 8x - 46 \geq 6 )</td>
</tr>
<tr>
<td>25.</td>
<td>( 4x^2 - 4x + 1 \leq 0 )</td>
</tr>
<tr>
<td>26.</td>
<td>( x^2 + 3x + 8 &gt; 0 )</td>
</tr>
</tbody>
</table>

In Exercises 27–32, solve the inequality and write the solution set in interval notation.

<table>
<thead>
<tr>
<th>Exercise</th>
<th>Inequality</th>
</tr>
</thead>
<tbody>
<tr>
<td>27.</td>
<td>( 4x^3 - 6x^2 &lt; 0 )</td>
</tr>
<tr>
<td>28.</td>
<td>( 4x^3 - 12x^2 &gt; 0 )</td>
</tr>
<tr>
<td>29.</td>
<td>( x^3 - 4x \geq 0 )</td>
</tr>
<tr>
<td>30.</td>
<td>( 2x^3 - x^4 \leq 0 )</td>
</tr>
<tr>
<td>31.</td>
<td>( (x - 1)^2(x + 2)^3 \geq 0 )</td>
</tr>
<tr>
<td>32.</td>
<td>( x^3(x - 3) \leq 0 )</td>
</tr>
</tbody>
</table>

Graphical Analysis In Exercises 33–36, use a graphing utility to graph the equation. Use the graph to approximate the values of \( x \) that satisfy each inequality.

<table>
<thead>
<tr>
<th>Exercise</th>
<th>Equation</th>
<th>Inequalities</th>
</tr>
</thead>
<tbody>
<tr>
<td>33.</td>
<td>( y = -x^2 + 2x + 3 )</td>
<td>(a) ( y \leq 0 ) (b) ( y \geq 3 )</td>
</tr>
<tr>
<td>34.</td>
<td>( y = \frac{1}{2}x^2 - 2x + 1 )</td>
<td>(a) ( y \leq 0 ) (b) ( y \geq 7 )</td>
</tr>
<tr>
<td>35.</td>
<td>( y = \frac{1}{2}x^3 - \frac{3}{4}x )</td>
<td>(a) ( y \geq 0 ) (b) ( y \leq 6 )</td>
</tr>
<tr>
<td>36.</td>
<td>( y = x^3 - x^2 - 16x + 16 )</td>
<td>(a) ( y \leq 0 ) (b) ( y \geq 36 )</td>
</tr>
</tbody>
</table>

In Exercises 37–50, solve the inequality and graph the solution on the real number line.

<table>
<thead>
<tr>
<th>Exercise</th>
<th>Inequality</th>
</tr>
</thead>
<tbody>
<tr>
<td>37.</td>
<td>( \frac{1}{x} - x &gt; 0 )</td>
</tr>
<tr>
<td>38.</td>
<td>( \frac{1}{x} - 4 &lt; 0 )</td>
</tr>
<tr>
<td>39.</td>
<td>( \frac{x + 6}{x + 1} - 2 &lt; 0 )</td>
</tr>
<tr>
<td>40.</td>
<td>( \frac{x + 12}{x + 2} - 3 \geq 0 )</td>
</tr>
<tr>
<td>41.</td>
<td>( \frac{3x - 5}{x - 5} &gt; 4 )</td>
</tr>
<tr>
<td>42.</td>
<td>( \frac{5 + 7x}{1 + 2x} &lt; 4 )</td>
</tr>
<tr>
<td>43.</td>
<td>( \frac{4}{x + 5} \geq \frac{1}{2x + 3} )</td>
</tr>
<tr>
<td>44.</td>
<td>( \frac{9}{x + 3} \leq \frac{4}{x + 3} )</td>
</tr>
<tr>
<td>45.</td>
<td>( \frac{1}{x - 3} \leq \frac{9}{4x + 3} )</td>
</tr>
<tr>
<td>46.</td>
<td>( \frac{1}{x} \geq \frac{1}{x + 3} )</td>
</tr>
<tr>
<td>47.</td>
<td>( \frac{x^2 + 2x}{x^2 - 9} \leq 0 )</td>
</tr>
<tr>
<td>48.</td>
<td>( \frac{x^2 + x - 6}{x} \geq 0 )</td>
</tr>
<tr>
<td>49.</td>
<td>( \frac{5}{x - 1} - \frac{2x}{x + 1} &lt; 1 )</td>
</tr>
<tr>
<td>50.</td>
<td>( \frac{3x}{x - 1} \leq \frac{x}{x + 4} + 3 )</td>
</tr>
</tbody>
</table>
Graphical Analysis  In Exercises 51–54, use a graphing utility to graph the equation. Use the graph to approximate the values of that satisfy each inequality.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Inequalities</th>
</tr>
</thead>
<tbody>
<tr>
<td>51. [ y = \frac{3x}{x - 2} ]</td>
<td>(a) [ y \leq 0 ] \hspace{1em} (b) [ y \geq 6 ]</td>
</tr>
<tr>
<td>52. [ y = \frac{2(x - 2)}{x + 1} ]</td>
<td>(a) [ y \leq 0 ] \hspace{1em} (b) [ y \geq 8 ]</td>
</tr>
<tr>
<td>53. [ y = \frac{2x^2}{x^2 + 4} ]</td>
<td>(a) [ y \geq 1 ] \hspace{1em} (b) [ y \leq 2 ]</td>
</tr>
<tr>
<td>54. [ y = \frac{5x}{x^2 + 4} ]</td>
<td>(a) [ y \geq 1 ] \hspace{1em} (b) [ y \leq 0 ]</td>
</tr>
</tbody>
</table>

In Exercises 55–60, find the domain of \( x \) in the expression. Use a graphing utility to verify your result.

| \[ \sqrt{4 - x^2} \] | 56. \[ \sqrt{x^2 - 4} \] |
| \[ \sqrt{x^2 - 7x + 12} \] | 57. \[ \sqrt{144 - 9x^2} \] |
| \[ \sqrt{\frac{x}{x^2 - 2x - 35}} \] | 58. \[ \sqrt{x^3 - 9} \] |

In Exercises 61–66, solve the inequality. (Round your answers to two decimal places.)

| \( 0.4x^2 + 5.26 < 10.2 \) | 61. \( -1.3x^2 + 3.78 > 2.12 \) |
| \( -0.5x^2 + 12.5x + 1.6 > 0 \) | 62. \( 1.2x^2 + 4.8x + 3.1 < 5.3 \) |
| \( \frac{1}{2.3x - 5.2} > 3.4 \) | 63. \( \frac{2}{3.1x - 3.7} > 5.8 \) |

Height of a Projectile  A projectile is fired straight upward from ground level with an initial velocity of 160 feet per second.

(a) At what instant will it be back at ground level?
(b) When will the height exceed 384 feet?

Height of a Projectile  A projectile is fired straight upward from ground level with an initial velocity of 128 feet per second.

(a) At what instant will it be back at ground level?
(b) When will the height be less than 128 feet?

Geometry  A rectangular playing field with a perimeter of 100 meters is to have an area of at least 500 square meters. Within what bounds must the length of the rectangle lie?

Geometry  A rectangular parking lot with a perimeter of 440 feet is to have an area of at least 8000 square feet. Within what bounds must the length of the rectangle lie?

Cost, Revenue, and Profit  The revenue and cost equations for a product are

\[ R = x(175 - 0.005x) \] \hspace{1em} \[ C = 30x + 250,000 \]

where \( R \) and \( C \) are measured in dollars and \( x \) represents the number of units sold. How many units must be sold to obtain a profit of at least $750,000? What is the price per unit?

Cost, Revenue, and Profit  The revenue and cost equations for a product are

\[ R = x(50 - 0.002x) \] \hspace{1em} \[ C = 12x + 150,000 \]

where \( R \) and \( C \) are measured in dollars and \( x \) represents the number of units sold. How many units must be sold to obtain a profit of at least $1,650,000? What is the price per unit?

Model It

Cable Television  The percents \( C \) of households in the United States that owned a television and had cable from 1980 to 2003 can be modeled by

\[ C = 0.0031t^2 - 0.216t^2 + 5.54t + 19.1, \]

\[ 0 \leq t \leq 23 \]

where \( t \) is the year, with \( t = 0 \) corresponding to 1980.

(a) Use a graphing utility to graph the equation.
(b) Complete the table to determine the year in which the percent of households that own a television and have cable will exceed 75%.
(c) Use the trace feature of a graphing utility to verify your answer to part (b).
(d) Complete the table to determine the years during which the percent of households that own a television and have cable will be between 85% and 100%.
(e) Use the trace feature of a graphing utility to verify your answer to part (d).
(f) Explain why the model may give values greater than 100% even though such values are not reasonable.
74. **Safe Load**  The maximum safe load uniformly distributed over a one-foot section of a two-inch-wide wooden beam is approximated by the model 
\[ \text{Load} = 168.5d^2 - 472.1, \]
where \( d \) is the depth of the beam.

(a) Evaluate the model for \( d = 4, d = 6, d = 8, d = 10, \) and \( d = 12 \). Use the results to create a bar graph.

(b) Determine the minimum depth of the beam that will safely support a load of 2000 pounds.

75. **Resistors**  When two resistors of resistances \( R_1 \) and \( R_2 \) are connected in parallel (see figure), the total resistance \( R \) satisfies the equation
\[ \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}. \]

Find \( R_2 \) for a parallel circuit in which \( R_2 = 2 \) ohms and \( R \) must be at least 1 ohm.

76. **Education**  The numbers \( N \) (in thousands) of master’s degrees earned by women in the United States from 1990 to 2002 are approximated by the model
\[ N = -0.03t^2 + 9.6t + 172 \]
where \( t \) represents the year, with \( t = 0 \) corresponding to 1990 (see figure). (Source: U.S. National Center for Education Statistics)

(a) According to the model, during what year did the number of master’s degrees earned by women exceed 220,000?

(b) Use the graph to verify the result of part (a).

(c) According to the model, during what year will the number of master’s degrees earned by women exceed 320,000?

(d) Use the graph to verify the result of part (c).

---

**Synthesis**

**True or False?**  In Exercises 77 and 78, determine whether the statement is true or false. Justify your answer.

77. The zeros of the polynomial \( x^3 - 2x^2 - 11x + 12 \) divide the real number line into four test intervals.

78. The solution set of the inequality \( \frac{1}{2}x^2 + 3x + 6 \geq 0 \) is the entire set of real numbers.

**Exploration**  In Exercises 79–82, find the interval for \( b \) such that the equation has at least one real solution.

79. \( x^2 + bx + 4 = 0 \)

80. \( x^2 + bx - 4 = 0 \)

81. \( 3x^2 + bx + 10 = 0 \)

82. \( 2x^2 + bx + 5 = 0 \)

83. (a) Write a conjecture about the intervals for \( b \) in Exercises 79–82. Explain your reasoning.

(b) What is the center of each interval for \( b \) in Exercises 79–82?

84. Consider the polynomial \( (x - a)(x - b) \) and the real number line shown below.

(a) Identify the points on the line at which the polynomial is zero.

(b) In each of the three subintervals of the line, write the sign of each factor and the sign of the product.

(c) For what \( x \)-values does the polynomial change signs?

**Skills Review**

In Exercises 85–88, factor the expression completely.

85. \( 4x^2 + 20x + 25 \)

86. \( (x + 3)^2 - 16 \)

87. \( x^2(x + 3) - 4(x + 3) \)

88. \( 2x^4 - 54x \)

In Exercises 89 and 90, write an expression for the area of the region.

89. \[
\begin{align*}
\text{Area} &= 2x + 1
\end{align*}
\]

90. \[
\begin{align*}
\text{Area} &= 3b^2 + 2
\end{align*}
\]