2.1 Quadratic Functions and Models

What you should learn

- Analyze graphs of quadratic functions.
- Write quadratic functions in standard form and use the results to sketch graphs of functions.
- Use quadratic functions to model and solve real-life problems.

Why you should learn it

Quadratic functions can be used to model data to analyze consumer behavior. For instance, in Exercise 83 on page 137, you will use a quadratic function to model the revenue earned from manufacturing handheld video games.

The *HM mathSpace*® CD-ROM and *Eduspace*® for this text contain additional resources related to the concepts discussed in this chapter.

The Graph of a Quadratic Function

In this and the next section, you will study the graphs of polynomial functions. In Section 1.6, you were introduced to the following basic functions.

$f(x) = ax + b$ Linear function

$f(x) = c$ Constant function

$f(x) = x^2$ Squaring function

These functions are examples of **polynomial functions.**

Definition of Polynomial Function

Let n be a nonnegative integer and let $a_n, a_{n-1}, \ldots, a_2, a_1, a_0$ be real numbers with $a_n \neq 0$. The function given by

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$$

is called a **polynomial function of x with degree n.**

Polynomial functions are classified by degree. For instance, a constant function has degree 0 and a linear function has degree 1. In this section, you will study second-degree polynomial functions, which are called **quadratic functions.**

For instance, each of the following functions is a quadratic function.

$$f(x) = x^2 + 6x + 2$$
$$g(x) = 2(x + 1)^2 - 3$$
$$h(x) = 9 + \tfrac{1}{4}x^2$$
$$k(x) = -3x^2 + 4$$
$$m(x) = (x - 2)(x + 1)$$

Note that the squaring function is a simple quadratic function that has degree 2.

Definition of Quadratic Function

Let a, b, and c be real numbers with $a \neq 0$. The function given by

$f(x) = ax^2 + bx + c$ Quadratic function

is called a **quadratic function.**

The graph of a quadratic function is a special type of "U"-shaped curve called a **parabola.** Parabolas occur in many real-life applications—especially those involving reflective properties of satellite dishes and flashlight reflectors. You will study these properties in Section 10.2.

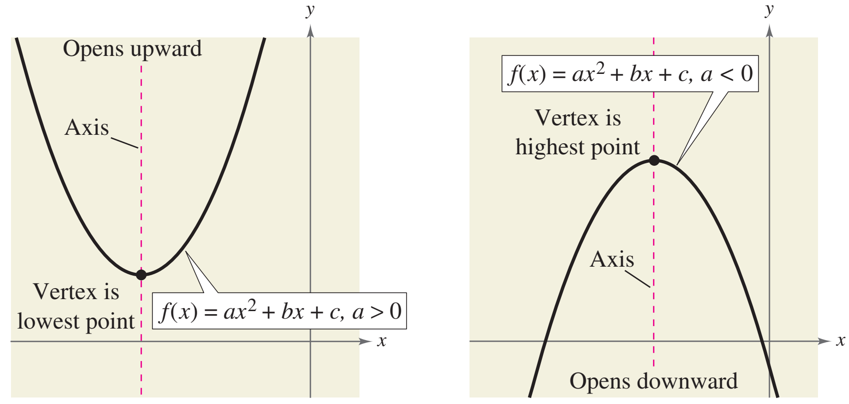
All parabolas are symmetric with respect to a line called the **axis of symmetry,** or simply the **axis** of the parabola. The point where the axis intersects the parabola is the **vertex** of the parabola, as shown in Figure 2.1. If the leading coefficient is positive, the graph of

$$f(x) = ax^2 + bx + c$$

is a parabola that opens upward. If the leading coefficient is negative, the graph of

$$f(x) = ax^2 + bx + c$$

is a parabola that opens downward.



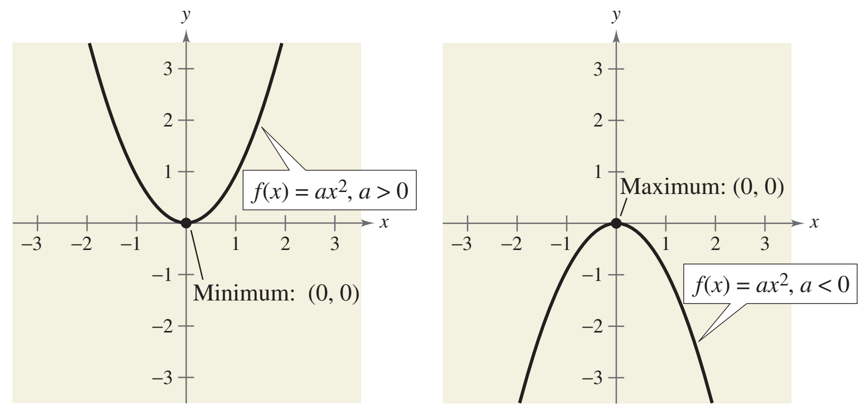
Leading coefficient is positive. *Leading coefficient is negative.*

FIGURE 2.1

The simplest type of quadratic function is

$$f(x) = ax^2.$$

Its graph is a parabola whose vertex is (0, 0). If $a > 0$, the vertex is the point with the *minimum* y-value on the graph, and if $a < 0$, the vertex is the point with the *maximum* y-value on the graph, as shown in Figure 2.2.



Leading coefficient is positive. *Leading coefficient is negative.*

FIGURE 2.2

When sketching the graph of $f(x) = ax^2$, it is helpful to use the graph of $y = x^2$ as a reference, as discussed in Section 1.7.

Exploration

Graph $y = ax^2$ for $a = -2, -1, -0.5, 0.5, 1$, and 2. How does changing the value of a affect the graph?

Graph $y = (x - h)^2$ for $h = -4, -2, 2$, and 4. How does changing the value of h affect the graph?

Graph $y = x^2 + k$ for $k = -4, -2, 2$, and 4. How does changing the value of k affect the graph?

Example 1 Sketching Graphs of Quadratic Functions

a. Compare the graphs of $y = x^2$ and $f(x) = \frac{1}{3}x^2$.

b. Compare the graphs of $y = x^2$ and $g(x) = 2x^2$.

Solution

a. Compared with $y = x^2$, each output of $f(x) = \frac{1}{3}x^2$ "shrinks" by a factor of $\frac{1}{3}$, creating the broader parabola shown in Figure 2.3.

b. Compared with $y = x^2$, each output of $g(x) = 2x^2$ "stretches" by a factor of 2, creating the narrower parabola shown in Figure 2.4.

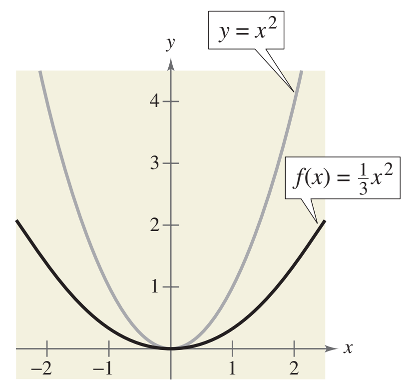


FIGURE 2.3

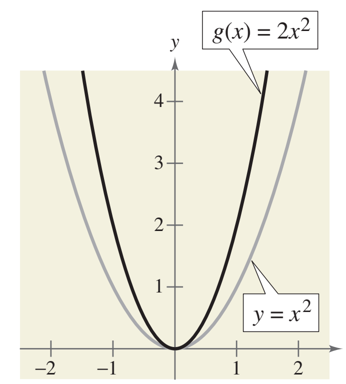


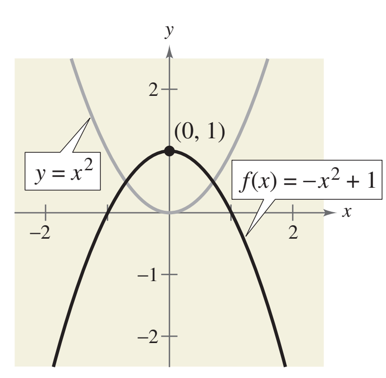
FIGURE 2.4

Emphasize the importance of comparing quadratic functions with the common function $y = x^2$, because it is very useful when analyzing functions and preparing to graph them.

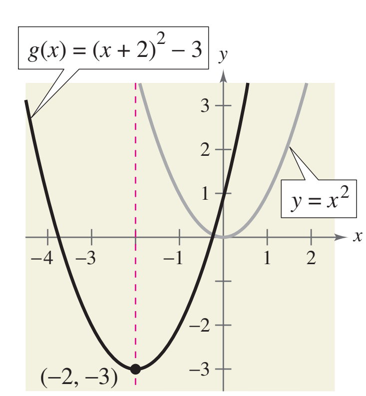
✓CHECKPOINT Now try Exercise 9.

In Example 1, note that the coefficient a determines how widely the parabola given by $f(x) = ax^2$ opens. If $|a|$ is small, the parabola opens more widely than if $|a|$ is large.

Recall from Section 1.7 that the graphs of $y = f(x \pm c)$, $y = f(x) \pm c$, $y = f(-x)$, and $y = -f(x)$ are rigid transformations of the graph of $y = f(x)$. For instance, in Figure 2.5, notice how the graph of $y = x^2$ can be transformed to produce the graphs of $f(x) = -x^2 + 1$ and $g(x) = (x + 2)^2 - 3$.



Reflection in x-axis followed by an upward shift of one unit



Left shift of two units followed by a downward shift of three units

FIGURE 2.5

The Standard Form of a Quadratic Function

The **standard form** of a quadratic function is $f(x) = a(x - h)^2 + k$. This form is especially convenient for sketching a parabola because it identifies the vertex of the parabola as (h, k).

Standard Form of a Quadratic Function

The quadratic function given by

$$f(x) = a(x - h)^2 + k, \qquad a \neq 0$$

is in **standard form.** The graph of f is a parabola whose axis is the vertical line $x = h$ and whose vertex is the point (h, k). If $a > 0$, the parabola opens upward, and if $a < 0$, the parabola opens downward.

STUDY TIP

The standard form of a quadratic function identifies four basic transformations of the graph of $y = x^2$.

a. The factor $|a|$ produces a vertical stretch or shrink.

b. If $a < 0$, the graph is reflected in the x-axis.

c. The factor $(x - h)^2$ represents a horizontal shift of h units.

d. The term k represents a vertical shift of k units.

To graph a parabola, it is helpful to begin by writing the quadratic function in standard form using the process of completing the square, as illustrated in Example 2. In this example, notice that when completing the square, you *add and subtract* the square of half the coefficient of x within the parentheses instead of adding the value to each side of the equation as is done in Appendix A.5.

To prepare for rewriting a quadratic-function in standard form, review the process of completing the square for an algebraic expression, paying special attention to problems in which $a \neq 1$.

Example 2 Graphing a Parabola in Standard Form

Sketch the graph of $f(x) = 2x^2 + 8x + 7$ and identify the vertex and the axis of the parabola.

Solution

Begin by writing the quadratic function in standard form. Notice that the first step in completing the square is to factor out any coefficient of x^2 that is not 1.

$$
\begin{aligned}
f(x) &= 2x^2 + 8x + 7 && \text{Write original function.}\\
&= 2(x^2 + 4x) + 7 && \text{Factor 2 out of } x\text{-terms.}\\
&= 2(x^2 + 4x + \underbrace{4}_{(4/2)^2} - 4) + 7 && \text{Add and subtract 4 within parentheses.}
\end{aligned}
$$

After adding and subtracting 4 within the parentheses, you must now regroup the terms to form a perfect square trinomial. The -4 can be removed from inside the parentheses; however, because of the 2 outside of the parentheses, you must multiply -4 by 2, as shown below.

$$
\begin{aligned}
f(x) &= 2(x^2 + 4x + 4) - 2(4) + 7 && \text{Regroup terms.}\\
&= 2(x^2 + 4x + 4) - 8 + 7 && \text{Simplify.}\\
&= 2(x + 2)^2 - 1 && \text{Write in standard form.}
\end{aligned}
$$

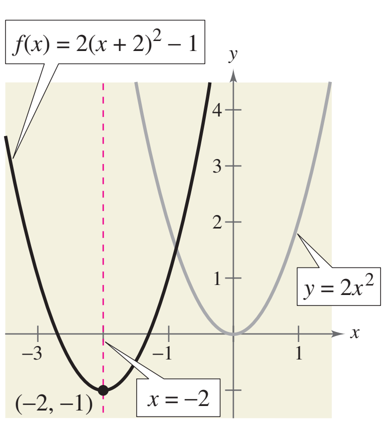


FIGURE 2.6

From this form, you can see that the graph of f is a parabola that opens upward and has its vertex at $(-2, -1)$. This corresponds to a left shift of two units and a downward shift of one unit relative to the graph of $y = 2x^2$, as shown in Figure 2.6. In the figure, you can see that the axis of the parabola is the vertical line through the vertex, $x = -2$.

CHECKPOINT Now try Exercise 13.

To find the x-intercepts of the graph of $f(x) = ax^2 + bx + c$, you must solve the equation $ax^2 + bx + c = 0$. If $ax^2 + bx + c$ does not factor, you can use the Quadratic Formula to find the x-intercepts. Remember, however, that a parabola may not have x-intercepts.

Example 3 Finding the Vertex and x-Intercepts of a Parabola

Sketch the graph of $f(x) = -x^2 + 6x - 8$ and identify the vertex and x-intercepts.

Solution

$$
\begin{aligned}
f(x) &= -x^2 + 6x - 8 && \text{Write original function.}\\
&= -(x^2 - 6x) - 8 && \text{Factor } -1 \text{ out of } x\text{-terms.}\\
&= -(x^2 - 6x + \underbrace{9}_{(-6/2)^2} - 9) - 8 && \text{Add and subtract 9 within parentheses.}\\
&= -(x^2 - 6x + 9) - (-9) - 8 && \text{Regroup terms.}\\
&= -(x - 3)^2 + 1 && \text{Write in standard form.}
\end{aligned}
$$

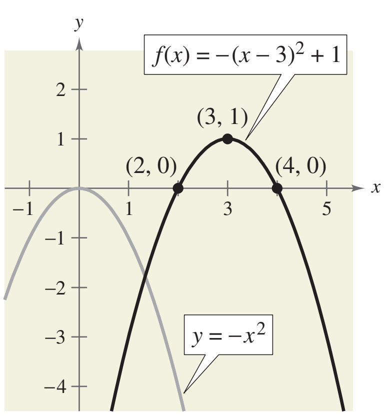


FIGURE 2.7

From this form, you can see that f is a parabola that opens downward with vertex $(3, 1)$. The x-intercepts of the graph are determined as follows.

$$
\begin{aligned}
-(x^2 - 6x + 8) &= 0 && \text{Factor out } -1.\\
-(x - 2)(x - 4) &= 0 && \text{Factor.}\\
x - 2 &= 0 \;\Rightarrow\; x = 2 && \text{Set 1st factor equal to 0.}\\
x - 4 &= 0 \;\Rightarrow\; x = 4 && \text{Set 2nd factor equal to 0.}
\end{aligned}
$$

So, the x-intercepts are $(2, 0)$ and $(4, 0)$, as shown in Figure 2.7.

CHECKPOINT Now try Exercise 19.

Example 4 Writing the Equation of a Parabola

Write the standard form of the equation of the parabola whose vertex is $(1, 2)$ and that passes through the point $(0, 0)$, as shown in Figure 2.8.

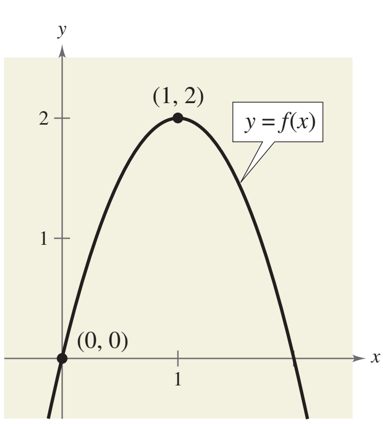


FIGURE 2.8

Solution

Because the vertex of the parabola is at $(h, k) = (1, 2)$, the equation has the form

$$f(x) = a(x - 1)^2 + 2. \qquad \text{Substitute for } h \text{ and } k \text{ in standard form.}$$

Because the parabola passes through the point $(0, 0)$, it follows that $f(0) = 0$. So,

$$0 = a(0 - 1)^2 + 2 \;\Rightarrow\; a = -2 \qquad \text{Substitute 0 for } x\text{; solve for } a.$$

which implies that the equation in standard form is $f(x) = -2(x - 1)^2 + 2$.

CHECKPOINT Now try Exercise 43.

Applications

Many applications involve finding the maximum or minimum value of a quadratic function. You can find the maximum or minimum value of a quadratic function by locating the vertex of the graph of the function.

You may wish to mention that much of beginning calculus deals with finding maxima and minima.

Activities

1. Describe the effect of a on the graph of $f(x) = ax^2 + bx + c$.
2. Rewrite the quadratic function $f(x) = -x^2 + 4x - 1$ in standard form.
   Answer: $f(x) = -(x - 2)^2 + 3$
3. Write the standard form of the equation of the parabola that has vertex $(3, -1)$ and passes through the point $(0, 2)$.
   Answer: $f(x) = \frac{1}{3}(x - 3)^2 - 1$
4. Fit a quadratic model to the points $(0, -2)$, $(2, 0)$, and $(-0.5, 0)$.
   Answer: $y = 2x^2 - 3x - 2$

Vertex of a Parabola

The vertex of the graph of $f(x) = ax^2 + bx + c$ is $\left(-\dfrac{b}{2a}, f\left(-\dfrac{b}{2a}\right)\right)$.

1. If $a > 0$, has a *minimum* at $x = -\dfrac{b}{2a}$.

2. If $a < 0$, has a *maximum* at $x = -\dfrac{b}{2a}$.

Example 5 The Maximum Height of a Baseball

A baseball is hit at a point 3 feet above the ground at a velocity of 100 feet per second and at an angle of 45° with respect to the ground. The path of the baseball is given by the function $f(x) = -0.0032x^2 + x + 3$, where $f(x)$ is the height of the baseball (in feet) and x is the horizontal distance from home plate (in feet). What is the maximum height reached by the baseball?

Solution

From the given function, you can see that $a = -0.0032$ and $b = 1$. Because the function has a maximum when $x = -b/(2a)$, you can conclude that the baseball reaches its maximum height when it is x feet from home plate, where x is

$$x = -\frac{b}{2a}x = -\frac{b}{2a} = -\frac{1}{2(-0.0032)} = 156.25 \text{ feet.}$$

At this distance, the maximum height is $f(156.25) = -0.0032(156.25)^2 + 156.25 + 3 = 81.125$ feet. The path of the baseball is shown in Figure 2.9.

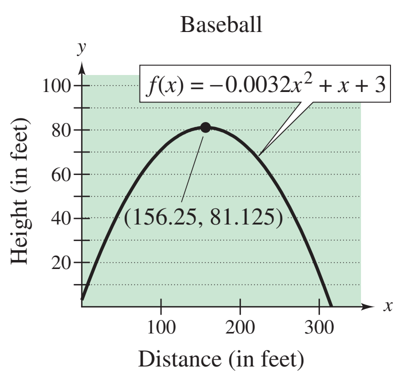


FIGURE 2.9

CHECKPOINT Now try Exercise 77.

Example 6 Minimizing Cost

A small local soft-drink manufacturer has daily production costs of $C = 70{,}000 - 120x + 0.075x^2$, where C is the total cost (in dollars) and x is the number of units produced. How many units should be produced each day to yield a minimum cost?

Solution

Use the fact that the function has a minimum when $x = -b/(2a)$. From the given function you can see that $a = 0.075$ and $b = -120$. So, producing

$$x = -\frac{b}{2a} = -\frac{-120}{2(0.075)} = 800 \text{ units}$$

each day will yield a minimum cost.

CHECKPOINT Now try Exercise 83.

2.1 Exercises

The *HM mathSpace*® CD-ROM and *Eduspace*® for this text contain step-by-step solutions to all odd-numbered exercises. They also provide Tutorial Exercises for additional help.

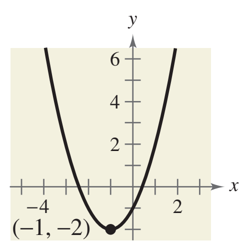
VOCABULARY CHECK: Fill in the blanks.

1. A polynomial function of degree n and leading coefficient a_n is a function of the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0\ (a_n \neq 0)$ where n is a ________ ________ and a_1 are ________ numbers.
2. A ________ function is a second-degree polynomial function, and its graph is called a ________.
3. The graph of a quadratic function is symmetric about its ________.
4. If the graph of a quadratic function opens upward, then its leading coefficient is ________ and the vertex of the graph is a ________.
5. If the graph of a quadratic function opens downward, then its leading coefficient is ________ and the vertex of the graph is a ________.

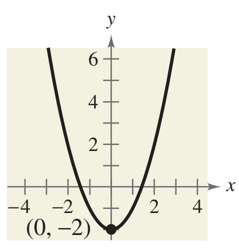
PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

In Exercises 1–8, match the quadratic function with its graph. [The graphs are labeled (a), (b), (c), (d), (e), (f), (g), and (h).]

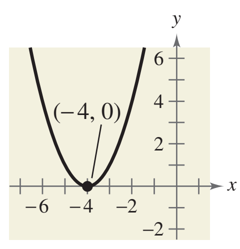
(a)



(b)



(c)



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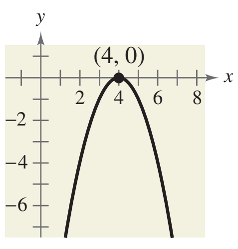

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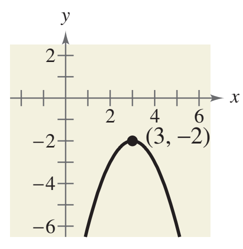
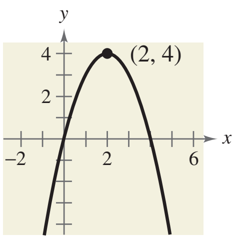
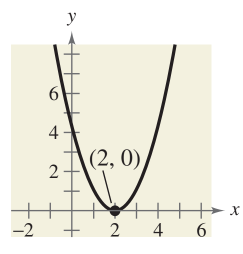


(f)



(g)



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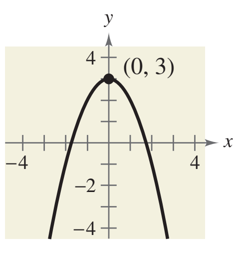


1. $f(x) = (x - 2)^2$ **2.** $f(x) = (x + 4)^2$

3. $f(x) = x^2 - 2$ **4.** $f(x) = 3 - x^2$

5. $f(x) = 4 - (x - 2)^2$ **6.** $f(x) = (x + 1)^2 - 2$

7. $f(x) = -(x - 3)^2 - 2$ **8.** $f(x) = -(x - 4)^2$

In Exercises 9–12, graph each function. Compare the graph of each function with the graph of $y = x^2$.

9. (a) $f(x) = \frac{1}{2}x^2$ (b) $g(x) = -\frac{1}{8}x^2$
(c) $h(x) = \frac{3}{2}x^2$ (d) $k(x) = -3x^2$

10. (a) $f(x) = x^2 + 1$ (b) $g(x) = x^2 - 1$
(c) $h(x) = x^2 + 3$ (d) $k(x) = x^2 - 3$

11. (a) $f(x) = (x - 1)^2$ (b) $g(x) = (3x)^2 + 1$
(c) $h(x) = \left(\frac{1}{3}x\right)^2 - 3$ (d) $k(x) = (x + 3)^2$

12. (a) $f(x) = -\frac{1}{2}(x - 2)^2 + 1$
(b) $g(x) = \left[\frac{1}{2}(x - 1)\right]^2 - 3$
(c) $h(x) = -\frac{1}{2}(x + 2)^2 - 1$
(d) $k(x) = [2(x + 1)]^2 + 4$

In Exercises 13–28, sketch the graph of the quadratic function without using a graphing utility. Identify the vertex, axis of symmetry, and x-intercept(s).

13. $f(x) = x^2 - 5$ **14.** $h(x) = 25 - x^2$

15. $f(x) = \frac{1}{2}x^2 - 4$ **16.** $f(x) = 16 - \frac{1}{4}x^2$

17. $f(x) = (x + 5)^2 - 6$ **18.** $f(x) = (x - 6)^2 + 3$

19. $h(x) = x^2 - 8x + 16$ **20.** $g(x) = x^2 + 2x + 1$

21. $f(x) = x^2 - x + \frac{5}{4}$ **22.** $f(x) = x^2 + 3x + \frac{1}{4}$

23. $f(x) = -x^2 + 2x + 5$ **24.** $f(x) = -x^2 - 4x + 1$

25. $h(x) = 4x^2 - 4x + 21$

26. $f(x) = 2x^2 - x + 1$

27. $f(x) = \frac{1}{4}x^2 - 2x - 12$

28. $f(x) = -\frac{1}{3}x^2 + 3x - 6$

In Exercises 29–36, use a graphing utility to graph the quadratic function. Identify the vertex, axis of symmetry, and x-intercepts. Then check your results algebraically by writing the quadratic function in standard form.

29. $f(x) = -(x^2 + 2x - 3)$ **30.** $f(x) = -(x^2 + x - 30)$

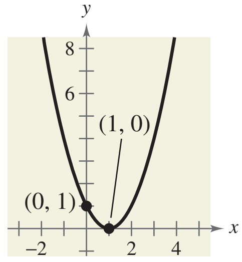
31. $g(x) = x^2 + 8x + 11$ **32.** $f(x) = x^2 + 10x + 14$

33. $f(x) = 2x^2 - 16x + 31$ **34.** $f(x) = -4x^2 + 24x - 41$

35. $g(x) = \frac{1}{2}(x^2 + 4x - 2)$ **36.** $f(x) = \frac{3}{5}(x^2 + 6x - 5)$

In Exercises 37–42, find the standard form of the quadratic function.

37.



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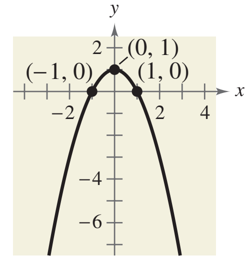

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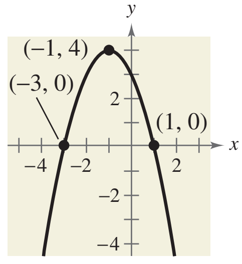

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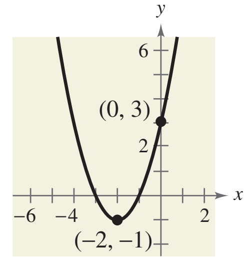

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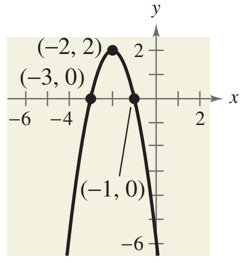

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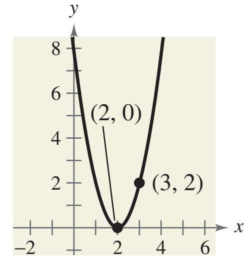


In Exercises 43–52, write the standard form of the equation of the parabola that has the indicated vertex and whose graph passes through the given point.

43. Vertex: $(-2, 5)$; point: $(0, 9)$

44. Vertex: $(4, -1)$; point: $(2, 3)$

45. Vertex: $(3, 4)$; point: $(1, 2)$

46. Vertex: $(2, 3)$; point: $(0, 2)$

47. Vertex: $(5, 12)$; point: $(7, 15)$

48. Vertex: $(-2, -2)$; point: $(-1, 0)$

49. Vertex: $\left(-\frac{1}{4}, \frac{3}{2}\right)$; point: $(-2, 0)$

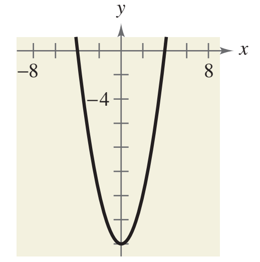
50. Vertex: $\left(\frac{5}{2}, -\frac{3}{4}\right)$; point: $(-2, 4)$

51. Vertex: $\left(-\frac{5}{2}, 0\right)$; point: $\left(-\frac{7}{2}, -\frac{16}{3}\right)$

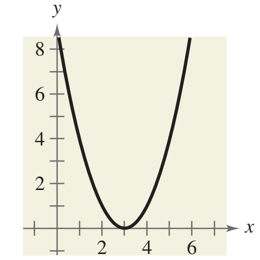
52. Vertex: $(6, 6)$; point: $\left(\frac{61}{10}, \frac{3}{2}\right)$

Graphical Reasoning **In Exercises 53–56, determine the x-intercept(s) of the graph visually. Then find the x-intercepts algebraically to confirm your results.**

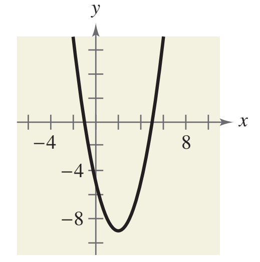
53. $y = x^2 - 16$



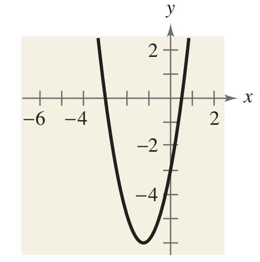
54. $y = x^2 - 6x + 9$



55. $y = x^2 - 4x - 5$



56. $y = 2x^2 + 5x - 3$



In Exercises 57–64, use a graphing utility to graph the quadratic function. Find the x-intercepts of the graph and compare them with the solutions of the corresponding quadratic equation when $f(x) = 0$.

57. $f(x) = x^2 - 4x$

58. $f(x) = -2x^2 + 10x$

59. $f(x) = x^2 - 9x + 18$

60. $f(x) = x^2 - 8x - 20$

61. $f(x) = 2x^2 - 7x - 30$

62. $f(x) = 4x^2 + 25x - 21$

63. $f(x) = -\frac{1}{2}(x^2 - 6x - 7)$

64. $f(x) = \frac{7}{10}(x^2 + 12x - 45)$

In Exercises 65–70, find two quadratic functions, one that opens upward and one that opens downward, whose graphs have the given x-intercepts. (There are many correct answers.)

65. $(-1, 0), (3, 0)$ **66.** $(-5, 0), (5, 0)$

67. $(0, 0), (10, 0)$ **68.** $(4, 0), (8, 0)$

69. $(-3, 0), \left(-\frac{1}{2}, 0\right)$ **70.** $\left(-\frac{5}{2}, 0\right), (2, 0)$

In Exercises 71–74, find two positive real numbers whose product is a maximum.

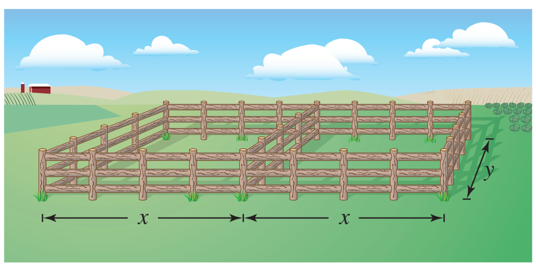
71. The sum is 110.

72. The sum is S.

73. The sum of the first and twice the second is 24.

74. The sum of the first and three times the second is 42.

75. ***Numerical, Graphical, and Analytical Analysis*** A rancher has 200 feet of fencing to enclose two adjacent rectangular corrals (see figure).



(a) Write the area A of the corral as a function of x.

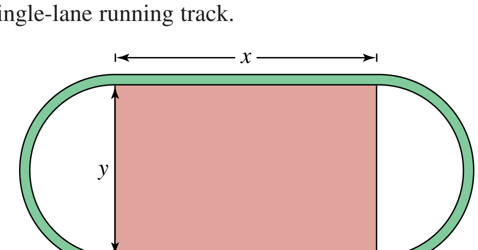
(b) Create a table showing possible values of x and the corresponding areas of the corral. Use the table to estimate the dimensions that will produce the maximum enclosed area.

(c) Use a graphing utility to graph the area function. Use the graph to approximate the dimensions that will produce the maximum enclosed area.

(d) Write the area function in standard form to find analytically the dimensions that will produce the maximum area.

(e) Compare your results from parts (b), (c), and (d).

76. ***Geometry*** An indoor physical fitness room consists of a rectangular region with a semicircle on each end (see figure). The perimeter of the room is to be a 200-meter single-lane running track.



(a) Determine the radius of the semicircular ends of the room. Determine the distance, in terms of y, around the inside edge of the two semicircular parts of the track.

(b) Use the result of part (a) to write an equation, in terms of x and y, for the distance traveled in one lap around the track. Solve for y.

(c) Use the result of part (b) to write the area A of the rectangular region as a function of x. What dimensions will produce a maximum area of the rectangle?

77. ***Path of a Diver*** The path of a diver is given by

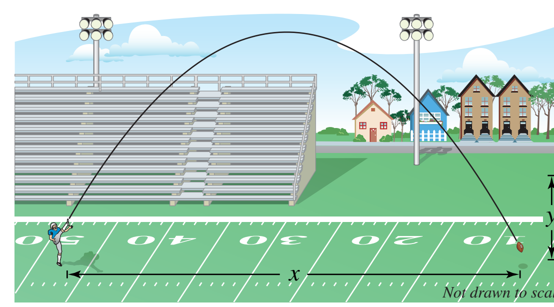
$$y = -\frac{4}{9}x^2 + \frac{24}{9}x + 12$$

where y is the height (in feet) and x is the horizontal distance from the end of the diving board (in feet). What is the maximum height of the diver?

78. ***Height of a Ball*** The height y (in feet) of a punted football is given by

$$y = -\frac{16}{2025}x^2 + \frac{9}{5}x + 1.5$$

where x is the horizontal distance (in feet) from the point at which the ball is punted (see figure).



(a) How high is the ball when it is punted?

(b) What is the maximum height of the punt?

(c) How long is the punt?

79. ***Minimum Cost*** A manufacturer of lighting fixtures has daily production costs of

$$C = 800 - 10x + 0.25x^2$$

where C is the total cost (in dollars) and x is the number of units produced. How many fixtures should be produced each day to yield a minimum cost?

80. ***Minimum Cost*** A textile manufacturer has daily production costs of

$$C = 100{,}000 - 110x + 0.045x^2$$

where C is the total cost (in dollars) and x is the number of units produced. How many units should be produced each day to yield a minimum cost?

81. ***Maximum Profit*** The profit P (in dollars) for a company that produces antivirus and system utilities software is

$$P = -0.0002x^2 + 140x - 250{,}000$$

where x is the number of units sold. What sales level will yield a maximum profit?

82. ***Maximum Profit*** The profit P (in hundreds of dollars) that a company makes depends on the amount x (in hundreds of dollars) the company spends on advertising according to the model

$$P = 230 + 20x - 0.5x^2.$$

What expenditure for advertising will yield a maximum profit?

83. ***Maximum Revenue*** The total revenue R earned (in thousands of dollars) from manufacturing handheld video games is given by

$$R(p) = -25p^2 + 1200p$$

where p is the price per unit (in dollars).

(a) Find the revenue earned for each price per unit given below.

$20

$25

$30

(b) Find the unit price that will yield a maximum revenue. What is the maximum revenue? Explain your results.

84. ***Maximum Revenue*** The total revenue R earned per day (in dollars) from a pet-sitting service is given by

$$R(p) = -12p^2 + 150p$$

where p is the price charged per pet (in dollars).

(a) Find the revenue earned for each price per pet given below.

$4

$6

$8

(b) Find the price that will yield a maximum revenue. What is the maximum revenue? Explain your results.

85. ***Graphical Analysis*** From 1960 to 2003, the per capita consumption C of cigarettes by Americans (age 18 and older) can be modeled by

$$C = 4299 - 1.8t - 1.36t^2, \quad 0 \le t \le 43$$

where t is the year, with $t = 0$ corresponding to 1960. (Source: *Tobacco Outlook Report*)

(a) Use a graphing utility to graph the model.

(b) Use the graph of the model to approximate the maximum average annual consumption. Beginning in 1966, all cigarette packages were required by law to carry a health warning. Do you think the warning had any effect? Explain.

(c) In 2000, the U.S. population (age 18 and over) was 209,128,094. Of those, about 48,308,590 were smokers. What was the average annual cigarette consumption *per smoker* in 2000? What was the average daily cigarette consumption *per smoker*?

Model It

86. ***Data Analysis*** The numbers y (in thousands) of hairdressers and cosmetologists in the United States for the years 1994 through 2002 are shown in the table. (Source: U.S. Bureau of Labor Statistics)

Year	Number of hairdressers and cosmetologists, y
1994	753
1995	750
1996	737
1997	748
1998	763
1999	784
2000	820
2001	854
2002	908

(a) Use a graphing utility to create a scatter plot of the data. Let x represent the year, with $x = 4$ corresponding to 1994.

(b) Use the *regression* feature of a graphing utility to find a quadratic model for the data.

(c) Use a graphing utility to graph the model in the same viewing window as the scatter plot. How well does the model fit the data?

(d) Use the *trace* feature of the graphing utility to approximate the year in which the number of hairdressers and cosmetologists was the least.

(e) Verify your answer to part (d) algebraically.

(f) Use the model to predict the number of hairdressers and cosmetologists in 2008.

87. ***Wind Drag*** The number of horsepower y required to overcome wind drag on an automobile is approximated by

$$y = 0.002s^2 + 0.005s - 0.029, \quad 0 \le s \le 100$$

where s is the speed of the car (in miles per hour).

(a) Use a graphing utility to graph the function.

(b) Graphically estimate the maximum speed of the car if the power required to overcome wind drag is not to exceed 10 horsepower. Verify your estimate algebraically.

88. ***Maximum Fuel Economy*** A study was done to compare the speed x (in miles per hour) with the mileage y (in miles per gallon) of an automobile. The results are shown in the table. (Source: Federal Highway Administration)

Speed, x	Mileage, y
15	22.3
20	25.5
25	27.5
30	29.0
35	28.8
40	30.0
45	29.9
50	30.2
55	30.4
60	28.8
65	27.4
70	25.3
75	23.3

(a) Use a graphing utility to create a scatter plot of the data.

(b) Use the *regression* feature of a graphing utility to find a quadratic model for the data.

(c) Use a graphing utility to graph the model in the same viewing window as the scatter plot.

(d) Estimate the speed for which the miles per gallon is greatest.

Synthesis

True or False? **In Exercises 89 and 90, determine whether the statement is true or false. Justify your answer.**

89. The function given by $f(x) = -12x^2 - 1$ has no x-intercepts.

90. The graphs of

$$f(x) = -4x^2 - 10x + 7$$

and

$$g(x) = 12x^2 + 30x + 1$$

have the same axis of symmetry.

91. Write the quadratic function

$$f(x) = ax^2 + bx + c$$

in standard form to verify that the vertex occurs at

$$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right).$$

92. ***Profit*** The profit P (in millions of dollars) for a recreational vehicle retailer is modeled by a quadratic function of the form

$$P = at^2 + bt + c$$

where t represents the year. If you were president of the company, which of the models below would you prefer? Explain your reasoning.

(a) a is positive and $-b/(2a) \le t$.

(b) a is positive and $t \le -b/(2a)$.

(c) a is negative and $-b/(2a) \le t$.

(d) a is negative and $t \le -b/(2a)$.

93. Is it possible for a quadratic equation to have only one x-intercept? Explain.

94. Assume that the function given by

$$f(x) = ax^2 + bx + c, \quad a \ne 0$$

has two real zeros. Show that the x-coordinate of the vertex of the graph is the average of the zeros of f. (*Hint:* Use the Quadratic Formula.)

Skills Review

In Exercises 95–98, find the equation of the line in slope-intercept form that has the given characteristics.

95. Passes through the points $(-4, 3)$ and $(2, 1)$

96. Passes through the point $\left(\frac{7}{2}, 2\right)$ and has a slope of $\frac{3}{2}$

97. Passes through the point $(0, 3)$ and is perpendicular to the line $4x + 5y = 10$

98. Passes through the point $(-8, 4)$ and is parallel to the line $y = -3x + 2$

In Exercises 99–104, let $f(x) = 14x - 3$ and let $g(x) = 8x^2$. Find the indicated value.

99. $(f + g)(-3)$

100. $(g - f)(2)$

101. $(fg)\left(-\frac{4}{7}\right)$

102. $\left(\frac{f}{g}\right)(-1.5)$

103. $(f \circ g)(-1)$

104. $(g \circ f)(0)$

105. **Make a Decision** To work an extended application analyzing the height of a basketball after it has been dropped, visit this text's website at *college.hmco.com*.