1 Chapter Summary

What did you learn?

Section 1.1	Review Exercises
☐ Plot points on the Cartesian plane *(p. 2)*.	1–4
☐ Use the Distance Formula to find the distance between two points *(p. 4)*.	5–8
☐ Use the Midpoint Formula to find the midpoint of a line segment *(p. 5)*.	5–8
☐ Use a coordinate plane and geometric formulas to model and solve real-life problems *(p. 6)*.	9–14
Section 1.2	
☐ Sketch graphs of equations *(p. 14)*.	15–24
☐ Find x- and y-intercepts of graphs of equations *(p. 17)*.	25–28
☐ Use symmetry to sketch graphs of equations *(p. 18)*.	29–36
☐ Find equations of and sketch graphs of circles *(p. 20)*.	37–44
☐ Use graphs of equations in solving real-life problems *(p. 21)*.	45, 46
Section 1.3	
☐ Use slope to graph linear equations in two variables *(p. 25)*.	47–50
☐ Find slopes of lines *(p. 27)*.	51–54
☐ Write linear equations in two variables *(p. 29)*.	55–62
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Section 1.5	
☐ Use the Vertical Line Test for functions *(p. 54)*.	81–84
☐ Find the zeros of functions *(p. 56)*.	85–88
☐ Determine intervals on which functions are increasing or decreasing and determine relative maximum and relative minimum values of functions *(p. 57)*.	89–94
☐ Determine the average rate of change of a function *(p. 59)*.	95–98
☐ Identify even and odd functions *(p. 60)*.	99–102

Section 1.6

- ☐ Identify and graph linear, squaring *(p. 66),* cubic, square root, reciprocal *(p. 68),* step, and other piecewise-defined functions *(p. 69).* — 103–114
- ☐ Recognize graphs of parent functions *(p. 70).* — 115, 116

Section 1.7

- ☐ Use vertical and horizontal shifts to sketch graphs of functions *(p. 74).* — 117–120
- ☐ Use reflections to sketch graphs of functions *(p. 76).* — 121–126
- ☐ Use nonrigid transformations to sketch graphs of functions *(p. 78).* — 127–130

Section 1.8

- ☐ Add, subtract, multiply, and divide functions *(p. 84).* — 131, 132
- ☐ Find the composition of one function with another function *(p. 86).* — 133–136
- ☐ Use combinations and compositions of functions to model and solve real-life problems *(p. 88).* — 137, 138

Section 1.9

- ☐ Find inverse functions informally and verify that two functions are inverse functions of each other *(p. 93).* — 139, 140
- ☐ Use graphs of functions to determine whether functions have inverse functions *(p. 95).* — 141, 142
- ☐ Use the Horizontal Line Test to determine if functions are one-to-one *(p. 96).* — 143–146
- ☐ Find inverse functions algebraically *(p. 97).* — 147–152

Section 1.10

- ☐ Use mathematical models to approximate sets of data points *(p. 103).* — 153
- ☐ Use the *regression* feature of a graphing utility to find the equation of a least squares regression line *(p. 104).* — 154
- ☐ Write mathematical models for direct variation *(p. 105).* — 155
- ☐ Write mathematical models for direct variation as an *n*th power *(p. 106).* — 156, 157
- ☐ Write mathematical models for inverse variation *(p. 107).* — 158, 159
- ☐ Write mathematical models for joint variation *(p. 108).* — 160

1 Review Exercises

1.1 In Exercises 1 and 2, plot the points in the Cartesian plane.

1. $(2, 2), (0, -4), (-3, 6), (-1, -7)$

2. $(5, 0), (8, 1), (4, -2), (-3, -3)$

In Exercises 3 and 4, determine the quadrant(s) in which (x, y) is located so that the condition(s) is (are) satisfied.

3. $x > 0$ and $y = -2$ **4.** $y > 0$

In Exercises 5–8, (a) plot the points, (b) find the distance between the points, and (c) find the midpoint of the line segment joining the points.

5. $(-3, 8), (1, 5)$

6. $(-2, 6), (4, -3)$

7. $(5.6, 0), (0, 8.2)$

8. $(0, -1.2), (-3.6, 0)$

In Exercises 9 and 10, the polygon is shifted to a new position in the plane. Find the coordinates of the vertices of the polygon in its new position.

9. Original coordinates of vertices:

$(4, 8), (6, 8), (4, 3), (6, 3)$

Shift: three units downward, two units to the left

10. Original coordinates of vertices:

$(0, 1), (3, 3), (0, 5), (-3, 3)$

Shift: five units upward, four units to the left

11. ***Sales*** The Cheesecake Factory had annual sales of \$539.1 million in 2001 and \$773.8 million in 2003. Use the Midpoint Formula to estimate the sales in 2002. (Source: The Cheesecake Factory, Inc.)

12. ***Meteorology*** The apparent temperature is a measure of relative discomfort to a person from heat and high humidity. The table shows the actual temperatures x (in degrees Fahrenheit) versus the apparent temperatures y (in degrees Fahrenheit) for a relative humidity of 75%.

x	70	75	80	85	90	95	100
y	70	77	85	95	109	130	150

(a) Sketch a scatter plot of the data shown in the table.

(b) Find the change in the apparent temperature when the actual temperature changes from 70°F to 100°F.

13. ***Geometry*** The volume of a globe is about 47,712.94 cubic centimeters. Find the radius of the globe.

14. ***Geometry*** The volume of a rectangular package is 2304 cubic inches. The length of the package is 3 times its width, and the height is 1.5 times its width.

(a) Draw a diagram that represents the problem. Label the height, width, and length accordingly.

(b) Find the dimensions of the package.

1.2 In Exercises 15–18, complete a table of values. Use the solution points to sketch the graph of the equation.

15. $y = 3x - 5$

16. $y = -\frac{1}{2}x + 2$

17. $y = x^2 - 3x$

18. $y = 2x^2 - x - 9$

In Exercises 19–24, sketch the graph *by hand*.

19. $y - 2x - 3 = 0$

20. $3x + 2y + 6 = 0$

21. $y = \sqrt{5 - x}$

22. $y = \sqrt{x + 2}$

23. $y + 2x^2 = 0$

24. $y = x^2 - 4x$

In Exercises 25–28, find the x- and y-intercepts of the graph of the equation.

25. $y = 2x + 7$

26. $y = |x + 1| - 3$

27. $y = (x - 3)^2 - 4$

28. $y = x\sqrt{4 - x^2}$

In Exercises 29–36, use the algebraic tests to check for symmetry with respect to both axes and the origin. Then sketch the graph of the equation.

29. $y = -4x + 1$

30. $y = 5x - 6$

31. $y = 5 - x^2$

32. $y = x^2 - 10$

33. $y = x^3 + 3$

34. $y = -6 - x^3$

35. $y = \sqrt{x + 5}$

36. $y = |x| + 9$

In Exercises 37–42, find the center and radius of the circle and sketch its graph.

37. $x^2 + y^2 = 9$

38. $x^2 + y^2 = 4$

39. $(x + 2)^2 + y^2 = 16$

40. $x^2 + (y - 8)^2 = 81$

41. $\left(x - \frac{1}{2}\right)^2 + (y + 1)^2 = 36$

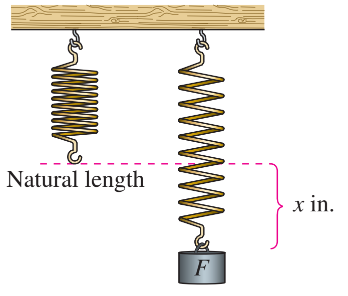
42. $(x + 4)^2 + \left(y - \frac{3}{2}\right)^2 = 100$

43. Find the standard form of the equation of the circle for which the endpoints of a diameter are $(0, 0)$ and $(4, -6)$.

44. Find the standard form of the equation of the circle for which the endpoints of a diameter are $(-2, -3)$ and $(4, -10)$.

45. ***Physics*** The force F (in pounds) required to stretch a spring x inches from its natural length (see figure) is

$$F = \frac{5}{4}x,\ 0 \le x \le 20.$$



(a) Use the model to complete the table.

x	0	4	8	12	16	20
Force, F						

(b) Sketch a graph of the model.

(c) Use the graph to estimate the force necessary to stretch the spring 10 inches.

46. ***Number of Stores*** The numbers N of Target stores for the years 1994 to 2003 can be approximated by the model

$$N = 3.69t^2 + 939, \quad 4 \le t \le 13$$

where t is the time (in years), with $t = 4$ corresponding to 1994. (Source: Target Corp.)

(a) Sketch a graph of the model.

(b) Use the graph to estimate the year in which the number of stores was 1300.

1.3 In Exercises 47–50, find the slope and *y*-intercept (if possible) of the equation of the line. Sketch the line.

47. $y = 6$

48. $x = -3$

49. $y = 3x + 13$

50. $y = -10x + 9$

In Exercises 51–54, plot the points and find the slope of the line passing through the pair of points.

51. $(3, -4), (-7, 1)$

52. $(-1, 8), (6, 5)$

53. $(-4.5, 6), (2.1, 3)$

54. $(-3, 2), (8, 2)$

In Exercises 55–58, find the slope-intercept form of the equation of the line that passes through the given point and has the indicated slope. Sketch the line.

	Point	*Slope*
55.	$(0, -5)$	$m = \frac{3}{2}$
56.	$(-2, 6)$	$m = 0$
57.	$(10, -3)$	$m = -\frac{1}{2}$
58.	$(-8, 5)$	m is undefined.

In Exercises 59–62, find the slope-intercept form of the equation of the line passing through the points.

59. $(0, 0), (0, 10)$

60. $(2, 5), (-2, -1)$

61. $(-1, 4), (2, 0)$

62. $(11, -2), (6, -1)$

In Exercises 63 and 64, write the slope-intercept forms of the equations of the lines through the given point (a) parallel to the given line and (b) perpendicular to the given line.

	Point	*Line*
63.	$(3, -2)$	$5x - 4y = 8$
64.	$(-8, 3)$	$2x + 3y = 5$

Rate of Change **In Exercises 65 and 66, you are given the dollar value of a product in 2006 and the rate at which the value of the product is expected to change during the next 5 years. Use this information to write a linear equation that gives the dollar value *V* of the product in terms of the year *t*. (Let $t = 6$ represent 2006.)**

	2006 Value	*Rate*
65.	\$12,500	\$850 increase per year
66.	\$72.95	\$5.15 increase per year

1.4 In Exercises 67–70, determine whether the equation represents y as a function of x.

67. $16x - y^4 = 0$

68. $2x - y - 3 = 0$

69. $y = \sqrt{1 - x}$

70. $|y| = x + 2$

In Exercises 71 and 72, evaluate the function at each specified value of the independent variable and simplify.

71. $f(x) = x^2 + 1$

(a) $f(2)$ (b) $f(-4)$ (c) $f(t^2)$ (d) $f(t + 1)$

72. $h(x) = \begin{cases} 2x + 1, & x \le -1 \\ x^2 + 2, & x > -1 \end{cases}$

(a) $h(-2)$ (b) $h(-1)$ (c) $h(0)$ (d) $h(2)$

In Exercises 73–76, find the domain of the function. Verify your result with a graph.

73. $f(x) = \sqrt{25 - x^2}$

74. $f(x) = 3x + 4$

75. $h(x) = \dfrac{x}{x^2 - x - 6}$

76. $h(t) = |t + 1|$

77. ***Physics*** The velocity of a ball projected upward from ground level is given by $v(t) = -32t + 48$, where t is the time in seconds and v is the velocity in feet per second.

(a) Find the velocity when $t = 1$.

(b) Find the time when the ball reaches its maximum height. [*Hint:* Find the time when $v(t) = 0$.]

(c) Find the velocity when $t = 2$.

78. ***Mixture Problem*** From a full 50-liter container of a 40% concentration of acid, x liters is removed and replaced with 100% acid.

(a) Write the amount of acid in the final mixture as a function of x.

(b) Determine the domain and range of the function.

(c) Determine x if the final mixture is 50% acid.

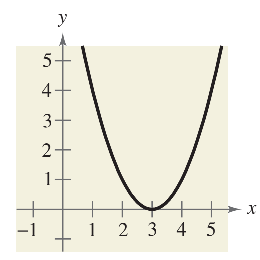
In Exercises 79 and 80, find the difference quotient and simplify your answer.

79. $f(x) = 2x^2 + 3x - 1, \quad \dfrac{f(x + h) - f(x)}{h}, \quad h \ne 0$

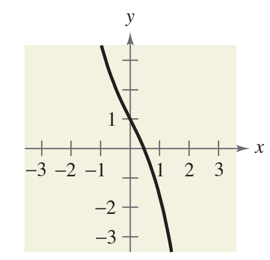
80. $f(x) = x^3 - 5x^2 + x, \quad \dfrac{f(x + h) - f(x)}{h}, \quad h \ne 0$

1.5 In Exercises 81–84, use the Vertical Line Test to determine whether y is a function of x. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

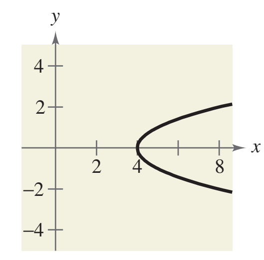
81. $y = (x - 3)^2$



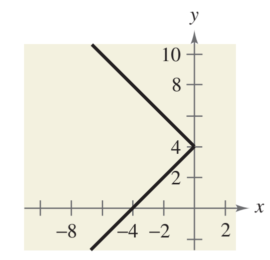
82. $y = -\frac{3}{5}x^3 - 2x + 1$



83. $x - 4 = y^2$



84. $x = -|4 - y|$



In Exercises 85–88, find the zeros of the function algebraically.

85. $f(x) = 3x^2 - 16x + 21$

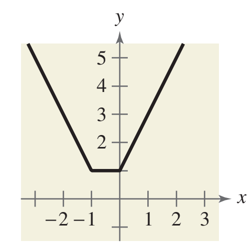
86. $f(x) = 5x^2 + 4x - 1$

87. $f(x) = \dfrac{8x + 3}{11 - x}$

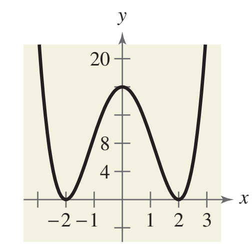
88. $f(x) = x^3 - x^2 - 25x + 25$

In Exercises 89 and 90, determine the intervals over which the function is increasing, decreasing, or constant.

89. $f(x) = |x| + |x + 1|$



90. $f(x) = (x^2 - 4)^2$



In Exercises 91–94, use a graphing utility to graph the function and approximate (to two decimal places) any relative minimum or relative maximum values.

91. $f(x) = -x^2 + 2x + 1$

92. $f(x) = x^4 - 4x^2 - 2$

93. $f(x) = x^3 - 6x^4$

94. $f(x) = x^3 - 4x^2 + x - 1$

In Exercises 95–98, find the average rate of change of the function from x_1 to x_2.

	Function	*x-Values*
95.	$f(x) = -x^2 + 8x - 4$	$x_1 = 0, x_2 = 4$
96.	$f(x) = x^3 + 12x - 2$	$x_1 = 0, x_2 = 4$
97.	$f(x) = 2 - \sqrt{x+1}$	$x_1 = 3, x_2 = 7$
98.	$f(x) = 1 - \sqrt{x+3}$	$x_1 = 1, x_2 = 6$

In Exercises 99–102, determine whether the function is even, odd, or neither.

99. $f(x) = x^5 + 4x - 7$

100. $f(x) = x^4 - 20x^2$

101. $f(x) = 2x\sqrt{x^2 + 3}$

102. $f(x) = \sqrt[5]{6x^2}$

1.6 In Exercises 103–104, write the linear function f such that it has the indicated function values. Then sketch the graph of the function.

103. $f(2) = -6,\ f(-1) = 3$

104. $f(0) = -5,\ f(4) = -8$

In Exercises 105–114, graph the function.

105. $f(x) = 3 - x^2$

106. $h(x) = x^3 - 2$

107. $f(x) = -\sqrt{x}$

108. $f(x) = \sqrt{x+1}$

109. $g(x) = \dfrac{3}{x}$

110. $g(x) = \dfrac{1}{x+5}$

111. $f(x) = [\![x]\!] - 2$

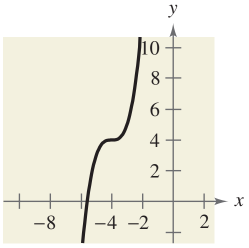
112. $g(x) = [\![x + 4]\!]$

113. $f(x) = \begin{cases} 5x - 3, & x \geq -1 \\ -4x + 5, & x < -1 \end{cases}$

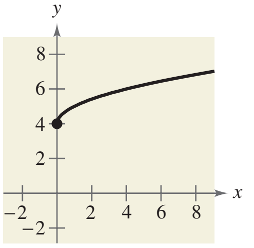
114. $f(x) = \begin{cases} x^2 - 2, & x < -2 \\ 5, & -2 \leq x \leq 0 \\ 8x - 5, & x > 0 \end{cases}$

In Exercises 115 and 116, the figure shows the graph of a transformed parent function. Identify the parent function.

115.



116.



1.7 In Exercises 117–130, h is related to one of the parent functions described in this chapter. (a) Identify the parent function f. (b) Describe the sequence of transformations from f to h. (c) Sketch the graph of h. (d) Use function notation to write h in terms of f.

117. $h(x) = x^2 - 9$

118. $h(x) = (x - 2)^3 + 2$

119. $h(x) = \sqrt{x - 7}$

120. $h(x) = |x + 3| - 5$

121. $h(x) = -(x + 3)^2 + 1$

122. $h(x) = -(x - 5)^3 - 5$

123. $h(x) = -[\![x]\!] + 6$

124. $h(x) = -\sqrt{x+1} + 9$

125. $h(x) = -|-x + 4| + 6$

126. $h(x) = -(x + 1)^2 - 3$

127. $h(x) = 5[\![x - 9]\!]$

128. $h(x) = -\frac{1}{3}x^3$

129. $h(x) = -2\sqrt{x - 4}$

130. $h(x) = \frac{1}{2}|x| - 1$

1.8 In Exercises 131 and 132, find (a) $(f + g)(x)$, (b) $(f - g)(x)$, (c) $(fg)(x)$, and (d) $(f/g)(x)$. What is the domain of f/g?

131. $f(x) = x^2 + 3,\quad g(x) = 2x - 1$

132. $f(x) = x^2 - 4,\quad g(x) = \sqrt{3 - x}$

In Exercises 133 and 134, find (a) $f \circ g$ and (b) $g \circ f$. Find the domain of each function and each composite function.

133. $f(x) = \frac{1}{3}x - 3,\quad g(x) = 3x + 1$

134. $f(x) = x^3 - 4,\quad g(x) = \sqrt[3]{x + 7}$

In Exercises 135 and 136, find two functions f and g such that $(f \circ g)(x) = h(x)$. (There are many correct answers.)

135. $h(x) = (6x - 5)^3$

136. $h(x) = \sqrt[3]{x + 2}$

137. ***Electronics Sales*** The factory sales (in millions of dollars) for VCRs $v(t)$ and DVD players $d(t)$ from 1997 to 2003 can be approximated by the functions

$$v(t) = -31.86t^2 + 233.6t + 2594$$

and

$$d(t) = -4.18t^2 + 571.0t - 3706$$

where t represents the year, with $t = 7$ corresponding to 1997. (Source: Consumer Electronics Association)

(a) Find and interpret $(v + d)(t)$.

(b) Use a graphing utility to graph $v(t)$, $d(t)$, and the function from part (a) in the same viewing window.

(c) Find $(v + d)(10)$. Use the graph in part (b) to verify your result.

138. ***Bacteria Count*** The number N of bacteria in a refrigerated food is given by

$$N(T) = 25T^2 - 50T + 300, \quad 2 \le T \le 20$$

where T is the temperature of the food in degrees Celsius. When the food is removed from refrigeration, the temperature of the food is given by

$$T(t) = 2t + 1, \quad 0 \le t \le 9$$

where t is the time in hours (a) Find the composition $N(T(t))$, and interpret its meaning in context, and (b) find the time when the bacterial count reaches 750.

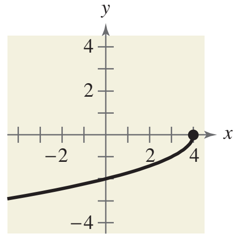
1.9 **In Exercises 139 and 140, find the inverse function of f informally. Verify that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.**

139. $f(x) = x - 7$

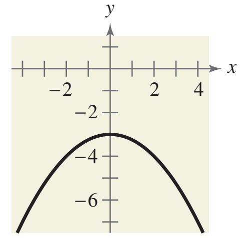
140. $f(x) = x + 5$

In Exercises 141 and 142, determine whether the function has an inverse function.

141.



142.



In Exercises 143–146, use a graphing utility to graph the function, and use the Horizontal Line Test to determine whether the function is one-to-one and so has an inverse function.

143. $f(x) = 4 - \frac{1}{3}x$

144. $f(x) = (x - 1)^2$

145. $h(t) = \dfrac{2}{t - 3}$

146. $g(x) = \sqrt{x + 6}$

In Exercises 147–150, (a) find the inverse function of f, (b) graph both f and f^{-1} on the same set of coordinate axes, (c) describe the relationship between the graphs of f and f^{-1}, and (d) state the domains and ranges of f and f^{-1}.

147. $f(x) = \frac{1}{2}x - 3$

148. $f(x) = 5x - 7$

149. $f(x) = \sqrt{x + 1}$

150. $f(x) = x^3 + 2$

In Exercises 151 and 152, restrict the domain of the function f to an interval over which the function is increasing and determine f^{-1} over that interval.

151. $f(x) = 2(x - 4)^2$

152. $f(x) = |x - 2|$

1.10 **153.** ***Median Income*** The median incomes I (in thousands of dollars) for married-couple families in the United States from 1995 through 2002 are shown in the table. A linear model that approximates these data is

$$I = 2.09t + 37.2$$

where t represents the year, with $t = 5$ corresponding to 1995. (Source: U.S. Census Bureau)

Year	Median income, I
1995	47.1
1996	49.7
1997	51.6
1998	54.2
1999	56.5
2000	59.1
2001	60.3
2002	61.1

(a) Plot the actual data and the model on the same set of coordinate axes.

(b) How closely does the model represent the data?

154. ***Data Analysis: Electronic Games*** The table shows the factory sales S (in millions of dollars) of electronic gaming software in the United States from 1995 through 2003. (Source: Consumer Electronics Association)

Year	Sales, S
1995	3000
1996	3500
1997	3900
1998	4480
1999	5100
2000	5850
2001	6725
2002	7375
2003	7744

(a) Use a graphing utility to create a scatter plot of the data. Let t represent the year, with $t = 5$ corresponding to 1995.

(b) Use the *regression* feature of the graphing utility to find the equation of the least squares regression line that fits the data. Then graph the model and the scatter plot you found in part (a) in the same viewing window. How closely does the model represent the data?

(c) Use the model to estimate the factory sales of electronic gaming software in the year 2008.

(d) Interpret the meaning of the slope of the linear model in the context of the problem.

155. ***Measurement*** You notice a billboard indicating that it is 2.5 miles or 4 kilometers to the next restaurant of a national fast-food chain. Use this information to find a mathematical model that relates miles to kilometers. Then use the model to find the numbers of kilometers in 2 miles and 10 miles.

156. ***Energy*** The power P produced by a wind turbine is proportional to the cube of the wind speed S. A wind speed of 27 miles per hour produces a power output of 750 kilowatts. Find the output for a wind speed of 40 miles per hour.

157. ***Frictional Force*** The frictional force F between the tires and the road required to keep a car on a curved section of a highway is directly proportional to the square of the speed s of the car. If the speed of the car is doubled, the force will change by what factor?

158. ***Demand*** A company has found that the daily demand x for its boxes of chocolates is inversely proportional to the price p. When the price is \$5, the demand is 800 boxes. Approximate the demand when the price is increased to \$6.

159. ***Travel Time*** The travel time between two cities is inversely proportional to the average speed. A train travels between the cities in 3 hours at an average speed of 65 miles per hour. How long would it take to travel between the cities at an average speed of 80 miles per hour?

160. ***Cost*** The cost of constructing a wooden box with a square base varies jointly as the height of the box and the square of the width of the box. A box of height 16 inches and width 6 inches costs \$28.80. How much would a box of height 14 inches and width 8 inches cost?

Synthesis

True or False? **In Exercises 161–163, determine whether the statement is true or false. Justify your answer.**

161. Relative to the graph of $f(x) = \sqrt{x}$, the function given by $h(x) = -\sqrt{x+9} - 13$ is shifted 9 units to the left and 13 units downward, then reflected in the x-axis.

162. If f and g are two inverse functions, then the domain of g is equal to the range of f.

163. If y is directly proportional to x, then x is directly proportional to y.

164. ***Writing*** Explain the difference between the Vertical Line Test and the Horizontal Line Test.

165. ***Writing*** Explain how to tell whether a relation between two variables is a function.

1 Chapter Test

Take this test as you would take a test in class. When you are finished, check your work against the answers given in the back of the book.

1. Plot the points $(-2, 5)$ and $(6, 0)$. Find the coordinates of the midpoint of the line segment joining the points and the distance between the points.

2. A cylindrical can has a volume of 600 cubic centimeters and a radius of 4 centimeters. Find the height of the can.

In Exercises 3–5, use intercepts and symmetry to sketch the graph of the equation.

3. $y = 3 - 5x$ **4.** $y = 4 - |x|$ **5.** $y = x^2 - 1$

6. Write the standard form of the equation of the circle shown at the left.

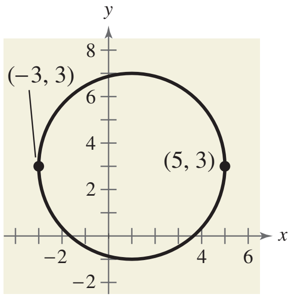


FIGURE FOR 6

In Exercises 7 and 8, find an equation of the line passing through the points.

7. $(2, -3), (-4, 9)$ **8.** $(3, 0.8), (7, -6)$

9. Find equations of the lines that pass through the point $(3, 8)$ and are (a) parallel to and (b) perpendicular to the line $-4x + 7y = -5$.

10. Evaluate $f(x) = \dfrac{\sqrt{x+9}}{x^2 - 81}$ at each value: (a) $f(7)$ (b) $f(-5)$ (c) $f(x - 9)$.

11. Determine the domain of $f(x) = \sqrt{100 - x^2}$.

In Exercises 12–14, (a) find the zeros of the function, (b) use a graphing utility to graph the function, (c) approximate the intervals over which the function is increasing, decreasing, or constant, and (d) determine whether the function is even, odd, or neither.

12. $f(x) = 2x^6 + 5x^4 - x^2$ **13.** $f(x) = 4x\sqrt{3 - x}$ **14.** $f(x) = |x + 5|$

15. Sketch the graph of $f(x) = \begin{cases} 3x + 7, & x \le -3 \\ 4x^2 - 1, & x > -3 \end{cases}$.

In Exercises 16 and 17, identify the parent function in the transformation. Then sketch a graph of the function.

16. $h(x) = -[\![x]\!]$ **17.** $h(x) = -\sqrt{x + 5} + 8$

In Exercises 18 and 19, find (a) $(f + g)(x)$, (b) $(f - g)(x)$, (c) $(fg)(x)$, (d) $(f/g)(x)$, (e) $(f \circ g)(x)$, and (f) $(g \circ f)(x)$.

18. $f(x) = 3x^2 - 7$, $g(x) = -x^2 - 4x + 5$ **19.** $f(x) = \dfrac{1}{x}$, $g(x) = 2\sqrt{x}$

In Exercises 20–22, determine whether or not the function has an inverse function, and if so, find the inverse function.

20. $f(x) = x^3 + 8$ **21.** $f(x) = |x^2 - 3| + 6$ **22.** $f(x) = 3x\sqrt{x}$

In Exercises 23–25, find a mathematical model representing the statement. (In each case, determine the constant of proportionality.)

23. v varies directly as the square root of s. ($v = 24$ when $s = 16$.)

24. A varies jointly as x and y. ($A = 500$ when $x = 15$ and $y = 8$.)

25. b varies inversely as a. ($b = 32$ when $a = 1.5$.)

Proofs in Mathematics

What does the word *proof* mean to you? In mathematics, the word *proof* is used to mean simply a valid argument. When you are proving a statement or theorem, you must use facts, definitions, and accepted properties in a logical order. You can also use previously proved theorems in your proof. For instance, the Distance Formula is used in the proof of the Midpoint Formula below. There are several different proof methods, which you will see in later chapters.

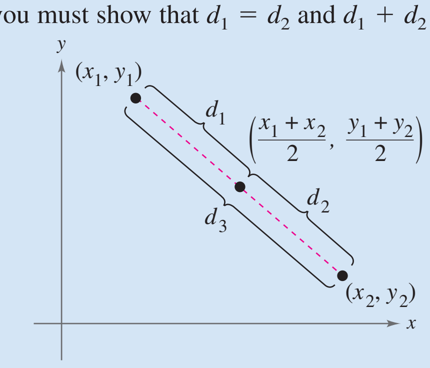
The Midpoint Formula *(p. 5)*

The midpoint of the line segment joining the points (x_1, y_1) and (x_2, y_2) is given by the Midpoint Formula

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Proof

Using the figure, you must show that $d_1 = d_2$ and $d_1 + d_2 = d_3$.



By the Distance Formula, you obtain

$$d_1 = \sqrt{\left(\frac{x_1 + x_2}{2} - x_1\right)^2 + \left(\frac{y_1 + y_2}{2} - y_1\right)^2}$$

$$= \frac{1}{2}\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d_2 = \sqrt{\left(x_2 - \frac{x_1 + x_2}{2}\right)^2 + \left(y_2 - \frac{y_1 + y_2}{2}\right)^2}$$

$$= \frac{1}{2}\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d_3 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

So, it follows that $d_1 = d_2$ and $d_1 + d_2 = d_3$.

The Cartesian Plane

The Cartesian plane was named after the French mathematician René Descartes (1596–1650). While Descartes was lying in bed, he noticed a fly buzzing around on the square ceiling tiles. He discovered that the position of the fly could be described by which ceiling tile the fly landed on. This led to the development of the Cartesian plane. Descartes felt that a coordinate plane could be used to facilitate description of the positions of objects.

P.S. Problem Solving

This collection of thought-provoking and challenging exercises further explores and expands upon concepts learned in this chapter.

1. As a salesperson, you receive a monthly salary of $2000, plus a commission of 7% of sales. You are offered a new job at $2300 per month, plus a commission of 5% of sales.

(a) Write a linear equation for your current monthly wage W_1 in terms of your monthly sales S.

(b) Write a linear equation for the monthly wage W_2 of your new job offer in terms of the monthly sales S.

(c) Use a graphing utility to graph both equations in the same viewing window. Find the point of intersection. What does it signify?

(d) You think you can sell $20,000 per month. Should you change jobs? Explain.

2. For the numbers 2 through 9 on a telephone keypad (see figure), create two relations: one mapping numbers onto letters, and the other mapping letters onto numbers. Are both relations functions? Explain.

3. What can be said about the sum and difference of each of the following?

(a) Two even functions (b) Two odd functions

(c) An odd function and an even function

4. The two functions given by

$f(x) = x$ and $g(x) = -x$

are their own inverse functions. Graph each function and explain why this is true. Graph other linear functions that are their own inverse functions. Find a general formula for a family of linear functions that are their own inverse functions.

5. Prove that a function of the following form is even.

$$y = a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdot \cdot \cdot + a_2x^2 + a_0$$

6. A miniature golf professional is trying to make a hole-in-one on the miniature golf green shown. A coordinate plane is placed over the golf green. The golf ball is at the point (2.5, 2) and the hole is at the point (9.5, 2). The professional wants to bank the ball off the side wall of the green at the point (x, y). Find the coordinates of the point (x, y). Then write an equation for the path of the ball.

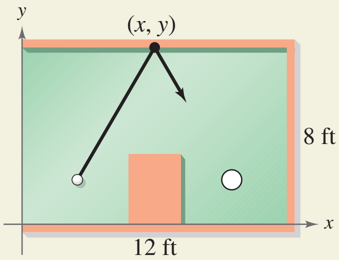


FIGURE FOR 6

7. At 2:00 P.M. on April 11, 1912, the *Titanic* left Cobh, Ireland, on her voyage to New York City. At 11:40 P.M. on April 14, the *Titanic* struck an iceberg and sank, having covered only about 2100 miles of the approximately 3400-mile trip.

(a) What was the total duration of the voyage in hours?

(b) What was the average speed in miles per hour?

(c) Write a function relating the distance of the *Titantic* from New York City and the number of hours traveled. Find the domain and range of the function.

(d) Graph the function from part (c).

8. Consider the function given by $f(x) = -x^2 + 4x - 3$. Find the average rate of change of the function from x_1 to x_2.

(a) $x_1 = 1, x_2 = 2$ (b) $x_1 = 1, x_2 = 1.5$

(c) $x_1 = 1, x_2 = 1.25$

(d) $x_1 = 1, x_2 = 1.125$

(e) $x_1 = 1, x_2 = 1.0625$

(f) Does the average rate of change seem to be approaching one value? If so, what value?

(g) Find the equations of the secant lines through the points $(x_1, f(x_1))$ and $(x_2, f(x_2))$ for parts (a)–(e).

(h) Find the equation of the line through the point $(1, f(1))$ using your answer from part (f) as the slope of the line.

9. Consider the functions given by $f(x) = 4x$ and $g(x) = x + 6$.

(a) Find $(f \circ g)(x)$.

(b) Find $(f \circ g)^{-1}(x)$.

(c) Find $f^{-1}(x)$ and $g^{-1}(x)$.

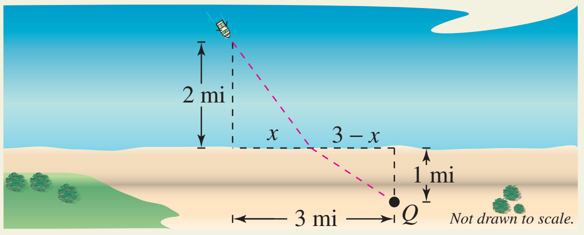
(d) Find $(g^{-1} \circ f^{-1})(x)$ and compare the result with that of part (b).

(e) Repeat parts (a) through (d) for $f(x) = x^3 + 1$ and $g(x) = 2x$.

(f) Write two one-to-one functions f and g, and repeat parts (a) through (d) for these functions.

(g) Make a conjecture about $(f \circ g)^{-1}(x)$ and $(g^{-1} \circ f^{-1})(x)$.

10. You are in a boat 2 miles from the nearest point on the coast. You are to travel to a point Q, 3 miles down the coast and 1 mile inland (see figure). You can row at 2 miles per hour and you can walk at 4 miles per hour.



(a) Write the total time T of the trip as a function of x.

(b) Determine the domain of the function.

(c) Use a graphing utility to graph the function. Be sure to choose an appropriate viewing window.

(d) Use the *zoom* and *trace* features to find the value of x that minimizes T.

(e) Write a brief paragraph interpreting these values.

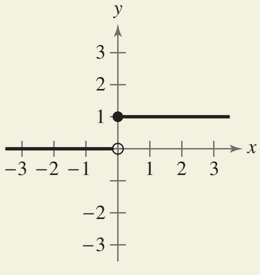
11. The **Heaviside function** $H(x)$ is widely used in engineering applications. (See figure.) To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

$$H(x) = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

Sketch the graph of each function by hand.

(a) $H(x) - 2$ (b) $H(x - 2)$ (c) $-H(x)$

(d) $H(-x)$ (e) $\frac{1}{2}H(x)$ (f) $-H(x - 2) + 2$



12. Let $f(x) = \dfrac{1}{1 - x}$.

(a) What are the domain and range of f?

(b) Find $f(f(x))$. What is the domain of this function?

(c) Find $f(f(f(x)))$. Is the graph a line? Why or why not?

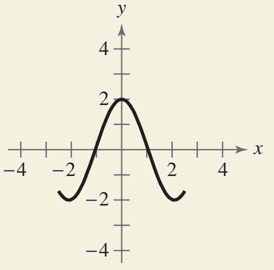
13. Show that the Associative Property holds for compositions of functions—that is,

$$(f \circ (g \circ h))(x) = ((f \circ g) \circ h)(x).$$

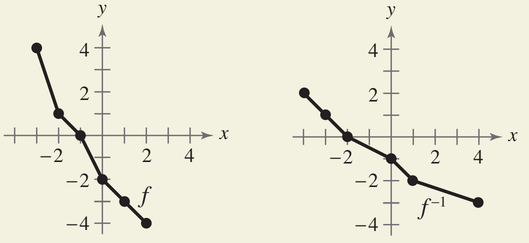
14. Consider the graph of the function f shown in the figure. Use this graph to sketch the graph of each function. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

(a) $f(x + 1)$ (b) $f(x) + 1$ (c) $2f(x)$ (d) $f(-x)$

(e) $-f(x)$ (f) $|f(x)|$ (g) $f(|x|)$



15. Use the graphs of f and f^{-1} to complete each table of function values.



(a)

x	-4	-2	0	4
$(f(f^{-1}(x))$				

(b)

x	-3	-2	0	1
$(f + f^{-1})(x)$				

(c)

x	-3	-2	0	1
$(f \cdot f^{-1})(x)$				

(d)

x	-4	-3	0	4
$\lvert f^{-1}(x)\rvert$				