

1.7 Transformations of Functions

What you should learn

- Use vertical and horizontal shifts to sketch graphs of functions.
- Use reflections to sketch graphs of functions.
- Use nonrigid transformations to sketch graphs of functions.

Why you should learn it

Knowing the graphs of common functions and knowing how to shift, reflect, and stretch graphs of functions can help you sketch a wide variety of simple functions by hand. This skill is useful in sketching graphs of functions that model real-life data, such as in Exercise 68 on page 83, where you are asked to sketch the graph of a function that models the amounts of mortgage debt outstanding from 1990 through 2002.



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STUDY TIP

In items 3 and 4, be sure you see that $h(x) = f(x - c)$ corresponds to a *right* shift and $h(x) = f(x + c)$ corresponds to a *left* shift for $c > 0$.

Shifting Graphs

Many functions have graphs that are simple transformations of the parent graphs summarized in Section 1.6. For example, you can obtain the graph of

$$h(x) = x^2 + 2$$

by shifting the graph of $f(x) = x^2$ *upward* two units, as shown in Figure 1.76. In function notation, h and f are related as follows.

$$h(x) = x^2 + 2 = f(x) + 2 \quad \text{Upward shift of two units}$$

Similarly, you can obtain the graph of

$$g(x) = (x - 2)^2$$

by shifting the graph of $f(x) = x^2$ to the *right* two units, as shown in Figure 1.77. In this case, the functions g and f have the following relationship.

$$g(x) = (x - 2)^2 = f(x - 2) \quad \text{Right shift of two units}$$

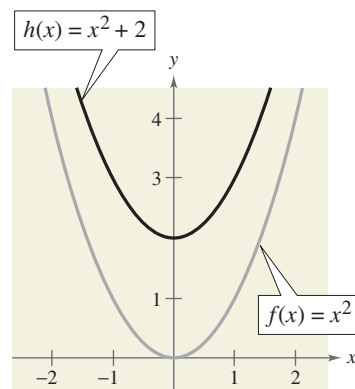


FIGURE 1.76

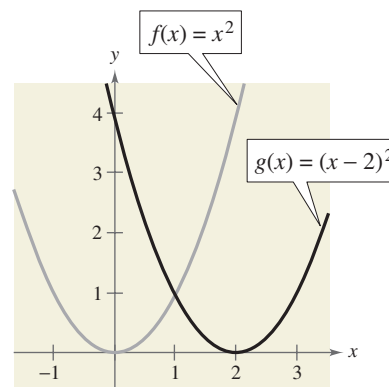


FIGURE 1.77

The following list summarizes this discussion about horizontal and vertical shifts.

Vertical and Horizontal Shifts

Let c be a positive real number. **Vertical and horizontal shifts** in the graph of $y = f(x)$ are represented as follows.

- | | |
|---|-------------------|
| 1. Vertical shift c units <i>upward</i> : | $h(x) = f(x) + c$ |
| 2. Vertical shift c units <i>downward</i> : | $h(x) = f(x) - c$ |
| 3. Horizontal shift c units to the <i>right</i> : | $h(x) = f(x - c)$ |
| 4. Horizontal shift c units to the <i>left</i> : | $h(x) = f(x + c)$ |

You might also wish to illustrate simple transformations of functions numerically using tables to emphasize what happens to individual ordered pairs. For instance, if you have $f(x) = x^2$, $h(x) = x^2 + 2 = f(x) + 2$, and $g(x) = (x - 2)^2 = f(x - 2)$, you can illustrate these transformations with the following tables.

| x | $f(x)$ | $h(x) = f(x) + 2$ |
|-----|--------|-------------------|
| -2 | 4 | $4 + 2 = 6$ |
| -1 | 1 | $1 + 2 = 3$ |
| 0 | 0 | $0 + 2 = 2$ |
| 1 | 1 | $1 + 2 = 3$ |
| 2 | 4 | $4 + 2 = 6$ |

| x | $x - 2$ | $g(x) = f(x - 2)$ |
|-----|--------------|-------------------|
| 0 | $0 - 2 = -2$ | 4 |
| 1 | $1 - 2 = -1$ | 1 |
| 2 | $2 - 2 = 0$ | 0 |
| 3 | $3 - 2 = 1$ | 1 |
| 4 | $4 - 2 = 2$ | 4 |

Some graphs can be obtained from combinations of vertical and horizontal shifts, as demonstrated in Example 1(b). Vertical and horizontal shifts generate a *family of functions*, each with the same shape but at different locations in the plane.

Example 1 Shifts in the Graphs of a Function

Use the graph of $f(x) = x^3$ to sketch the graph of each function.

- a. $g(x) = x^3 - 1$
- b. $h(x) = (x + 2)^3 + 1$

Solution

- a. Relative to the graph of $f(x) = x^3$, the graph of $g(x) = x^3 - 1$ is a downward shift of one unit, as shown in Figure 1.78.
- b. Relative to the graph of $f(x) = x^3$, the graph of $h(x) = (x + 2)^3 + 1$ involves a left shift of two units and an upward shift of one unit, as shown in Figure 1.79.

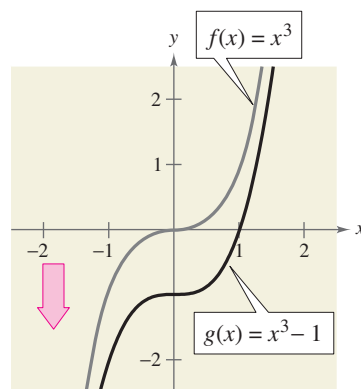


FIGURE 1.78

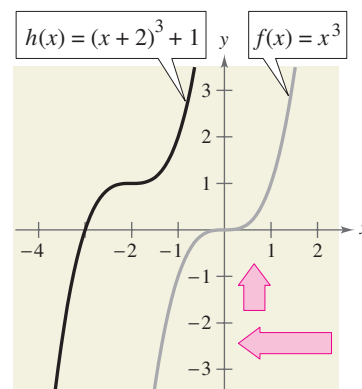


FIGURE 1.79

CHECKPOINT Now try Exercise 1.

In Figure 1.79, notice that the same result is obtained if the vertical shift precedes the horizontal shift *or* if the horizontal shift precedes the vertical shift.

Exploration

Graphing utilities are ideal tools for exploring translations of functions. Graph f , g , and h in same viewing window. Before looking at the graphs, try to predict how the graphs of g and h relate to the graph of f .

- a. $f(x) = x^2$, $g(x) = (x - 4)^2$, $h(x) = (x - 4)^2 + 3$
- b. $f(x) = x^2$, $g(x) = (x + 1)^2$, $h(x) = (x + 1)^2 - 2$
- c. $f(x) = x^2$, $g(x) = (x + 4)^2$, $h(x) = (x + 4)^2 + 2$

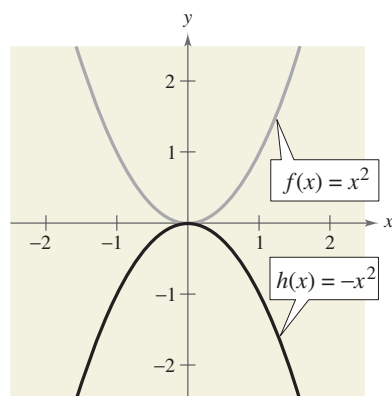


FIGURE 1.80

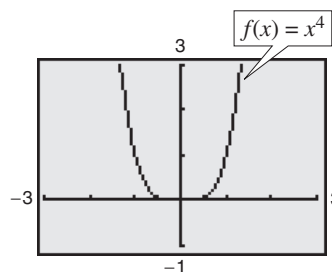


FIGURE 1.81

Reflecting Graphs

The second common type of transformation is a **reflection**. For instance, if you consider the x -axis to be a mirror, the graph of

$$h(x) = -x^2$$

is the mirror image (or reflection) of the graph of

$$f(x) = x^2,$$

as shown in Figure 1.80.

Reflections in the Coordinate Axes

Reflections in the coordinate axes of the graph of $y = f(x)$ are represented as follows.

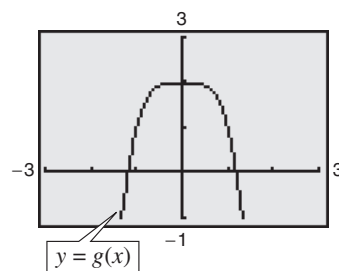
1. Reflection in the x -axis: $h(x) = -f(x)$
2. Reflection in the y -axis: $h(x) = f(-x)$

Example 2 Finding Equations from Graphs

The graph of the function given by

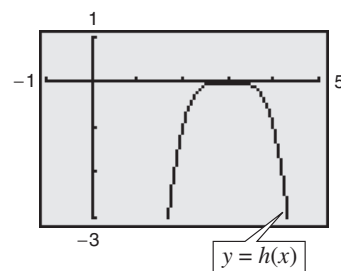
$$f(x) = x^4$$

is shown in Figure 1.81. Each of the graphs in Figure 1.82 is a transformation of the graph of f . Find an equation for each of these functions.



(a)

FIGURE 1.82



(b)

Exploration

Reverse the order of transformations in Example 2(a). Do you obtain the same graph? Do the same for Example 2(b). Do you obtain the same graph? Explain.

Solution

- a. The graph of g is a reflection in the x -axis followed by an upward shift of two units of the graph of $f(x) = x^4$. So, the equation for g is

$$g(x) = -x^4 + 2.$$

- b. The graph of h is a horizontal shift of three units to the right followed by a reflection in the x -axis of the graph of $f(x) = x^4$. So, the equation for h is

$$h(x) = -(x - 3)^4.$$



CHECKPOINT Now try Exercise 9.

Example 3 Reflections and Shifts

Compare the graph of each function with the graph of $f(x) = \sqrt{x}$.

a. $g(x) = -\sqrt{x}$ b. $h(x) = \sqrt{-x}$ c. $k(x) = -\sqrt{x+2}$

Algebraic Solution

- a. The graph of g is a reflection of the graph of f in the x -axis because

$$\begin{aligned} g(x) &= -\sqrt{x} \\ &= -f(x). \end{aligned}$$

- b. The graph of h is a reflection of the graph of f in the y -axis because

$$\begin{aligned} h(x) &= \sqrt{-x} \\ &= f(-x). \end{aligned}$$

- c. The graph of k is a left shift of two units followed by a reflection in the x -axis because

$$\begin{aligned} k(x) &= -\sqrt{x+2} \\ &= -f(x+2). \end{aligned}$$

Graphical Solution

- a. Graph f and g on the same set of coordinate axes. From the graph in Figure 1.83, you can see that the graph of g is a reflection of the graph of f in the x -axis.
- b. Graph f and h on the same set of coordinate axes. From the graph in Figure 1.84, you can see that the graph of h is a reflection of the graph of f in the y -axis.
- c. Graph f and k on the same set of coordinate axes. From the graph in Figure 1.85, you can see that the graph of k is a left shift of two units of the graph of f , followed by a reflection in the x -axis.

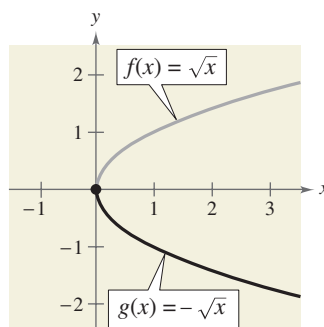


FIGURE 1.83

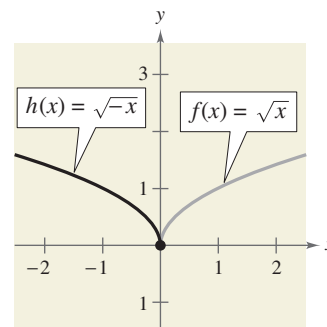


FIGURE 1.84

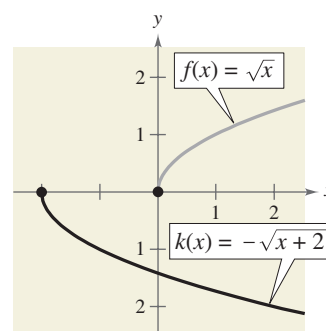


FIGURE 1.85



CHECKPOINT Now try Exercise 19.

Activities

1. How are the graphs of $f(x)$ and $g(x) = -f(x)$ related?

Answer: They are reflections of each other in the x -axis.

2. Compare the graph of $f(x) = |x|$ with the graph of $g(x) = |x-9|$.

Answer: $g(x)$ is $f(x)$ shifted to the right nine units.

When sketching the graphs of functions involving square roots, remember that the domain must be restricted to exclude negative numbers inside the radical. For instance, here are the domains of the functions in Example 3.

Domain of $g(x) = -\sqrt{x}$: $x \geq 0$

Domain of $h(x) = \sqrt{-x}$: $x \leq 0$

Domain of $k(x) = -\sqrt{x+2}$: $x \geq -2$

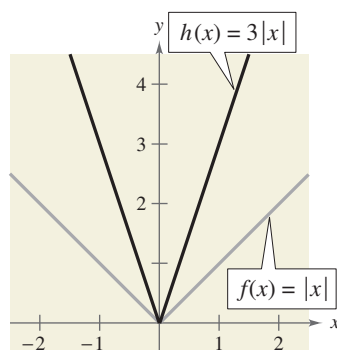


FIGURE 1.86

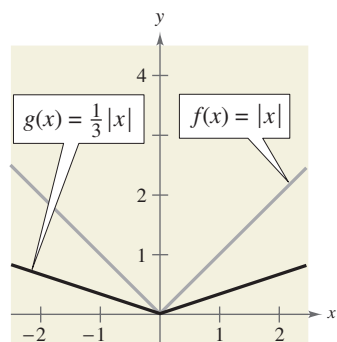


FIGURE 1.87

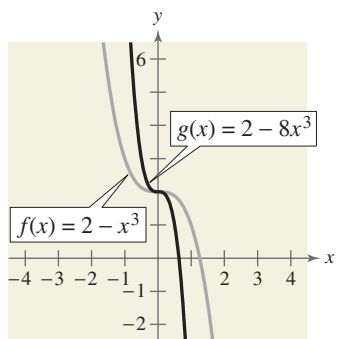


FIGURE 1.88

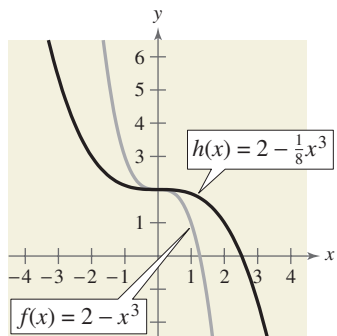


FIGURE 1.89

Nonrigid Transformations

Horizontal shifts, vertical shifts, and reflections are **rigid transformations** because the basic shape of the graph is unchanged. These transformations change only the *position* of the graph in the coordinate plane. **Nonrigid transformations** are those that cause a *distortion*—a change in the shape of the original graph. For instance, a nonrigid transformation of the graph of $y = f(x)$ is represented by $g(x) = cf(x)$, where the transformation is a **vertical stretch** if $c > 1$ and a **vertical shrink** if $0 < c < 1$. Another nonrigid transformation of the graph of $y = f(x)$ is represented by $h(x) = f(cx)$, where the transformation is a **horizontal shrink** if $c > 1$ and a **horizontal stretch** if $0 < c < 1$.

Example 4 Nonrigid Transformations

Compare the graph of each function with the graph of $f(x) = |x|$.

- a. $h(x) = 3|x|$ b. $g(x) = \frac{1}{3}|x|$

Solution

- a. Relative to the graph of $f(x) = |x|$, the graph of

$$h(x) = 3|x| = 3f(x)$$

is a vertical stretch (each y -value is multiplied by 3) of the graph of f . (See Figure 1.86.)

- b. Similarly, the graph of

$$g(x) = \frac{1}{3}|x| = \frac{1}{3}f(x)$$

is a vertical shrink (each y -value is multiplied by $\frac{1}{3}$) of the graph of f . (See Figure 1.87.)

CHECKPOINT Now try Exercise 23.

Example 5 Nonrigid Transformations

Compare the graph of each function with the graph of $f(x) = 2 - x^3$.

- a. $g(x) = f(2x)$ b. $h(x) = f(\frac{1}{2}x)$

Solution

- a. Relative to the graph of $f(x) = 2 - x^3$, the graph of

$$g(x) = f(2x) = 2 - (2x)^3 = 2 - 8x^3$$

is a horizontal shrink ($c > 1$) of the graph of f . (See Figure 1.88.)

- b. Similarly, the graph of

$$h(x) = f(\frac{1}{2}x) = 2 - (\frac{1}{2}x)^3 = 2 - \frac{1}{8}x^3$$

is a horizontal stretch ($0 < c < 1$) of the graph of f . (See Figure 1.89.)

CHECKPOINT Now try Exercise 27.

1.7 Exercises

VOCABULARY CHECK:

In Exercises 1–5, fill in the blanks.

- Horizontal shifts, vertical shifts, and reflections are called _____ transformations.
- A reflection in the x -axis of $y = f(x)$ is represented by $h(x) = \underline{\hspace{2cm}}$, while a reflection in the y -axis of $y = f(x)$ is represented by $h(x) = \underline{\hspace{2cm}}$.
- Transformations that cause a distortion in the shape of the graph of $y = f(x)$ are called _____ transformations.
- A nonrigid transformation of $y = f(x)$ represented by $h(x) = f(cx)$ is a _____ if $c > 1$ and a _____ if $0 < c < 1$.
- A nonrigid transformation of $y = f(x)$ represented by $g(x) = cf(x)$ is a _____ if $c > 1$ and a _____ if $0 < c < 1$.
- Match the rigid transformation of $y = f(x)$ with the correct representation of the graph of h , where $c > 0$.

| | |
|-----------------------|---|
| (a) $h(x) = f(x) + c$ | (i) A horizontal shift of f , c units to the right |
| (b) $h(x) = f(x) - c$ | (ii) A vertical shift of f , c units downward |
| (c) $h(x) = f(x + c)$ | (iii) A horizontal shift of f , c units to the left |
| (d) $h(x) = f(x - c)$ | (iv) A vertical shift of f , c units upward |

PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at www.Eduspace.com.

- For each function, sketch (on the same set of coordinate axes) a graph of each function for $c = -1, 1$, and 3 .

- $f(x) = |x| + c$
- $f(x) = |x - c|$
- $f(x) = |x + 4| + c$

- For each function, sketch (on the same set of coordinate axes) a graph of each function for $c = -3, -1, 1$, and 3 .

- $f(x) = \sqrt{x} + c$
- $f(x) = \sqrt{x - c}$
- $f(x) = \sqrt{x - 3} + c$

- For each function, sketch (on the same set of coordinate axes) a graph of each function for $c = -2, 0$, and 2 .

- $f(x) = \lfloor x \rfloor + c$
- $f(x) = \lfloor x + c \rfloor$
- $f(x) = \lfloor x - 1 \rfloor + c$

- For each function, sketch (on the same set of coordinate axes) a graph of each function for $c = -3, -1, 1$, and 3 .

- $f(x) = \begin{cases} x^2 + c, & x < 0 \\ -x^2 + c, & x \geq 0 \end{cases}$
- $f(x) = \begin{cases} (x + c)^2, & x < 0 \\ -(x + c)^2, & x \geq 0 \end{cases}$

In Exercises 5–8, use the graph of f to sketch each graph. To print an enlarged copy of the graph go to the website www.mathgraphs.com.

- $y = f(x) + 2$
 - $y = f(x - 2)$
 - $y = 2f(x)$
 - $y = -f(x)$
 - $y = f(x + 3)$
 - $y = f(-x)$
 - $y = f(\frac{1}{2}x)$

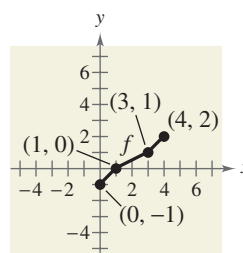


FIGURE FOR 5

- $y = f(x) - 1$
 - $y = f(x - 1)$
 - $y = f(-x)$
 - $y = f(x + 1)$
 - $y = -f(x - 2)$
 - $y = \frac{1}{2}f(x)$
 - $y = f(2x)$

- $y = f(-x)$
 - $y = f(x) + 4$
 - $y = 2f(x)$
 - $y = -f(x - 4)$
 - $y = f(x) - 3$
 - $y = -f(x) - 1$
 - $y = f(2x)$

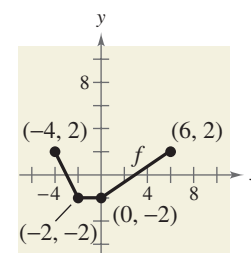


FIGURE FOR 6

- $y = f(x - 5)$
 - $y = -f(x) + 3$
 - $y = \frac{1}{3}f(x)$
 - $y = -f(x + 1)$
 - $y = f(-x)$
 - $y = f(x) - 10$
 - $y = f(\frac{1}{3}x)$

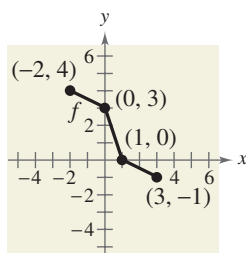
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FIGURE FOR 7

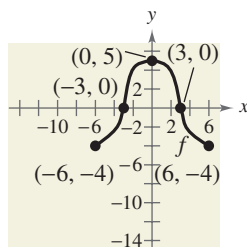
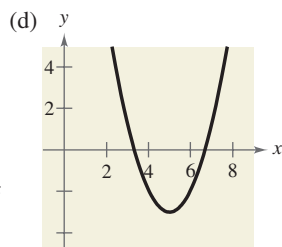
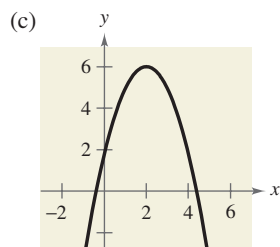
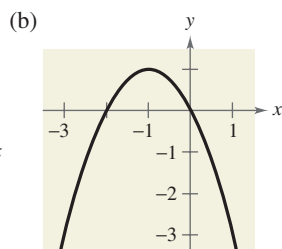
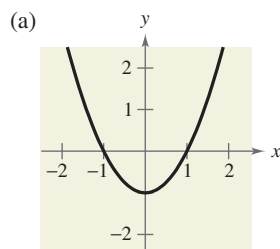
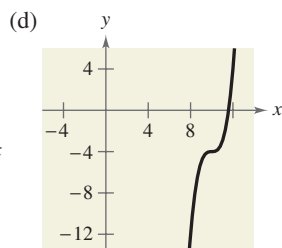
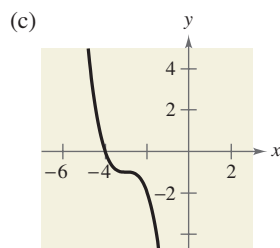
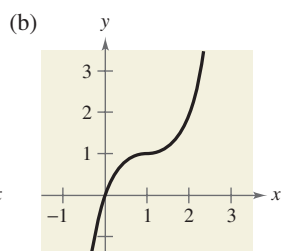
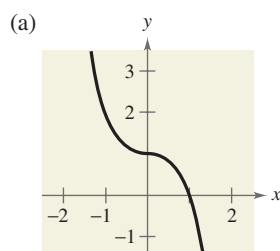


FIGURE FOR 8

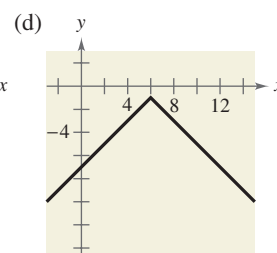
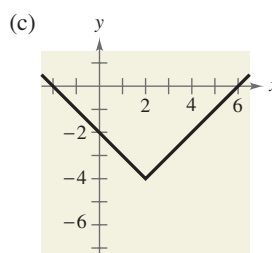
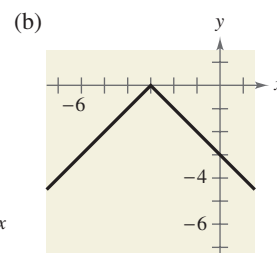
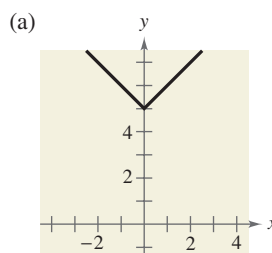
9. Use the graph of $f(x) = x^2$ to write an equation for each function whose graph is shown.



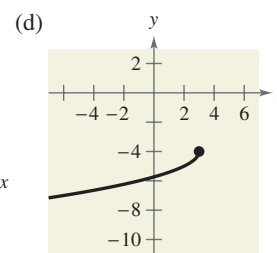
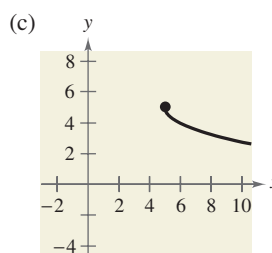
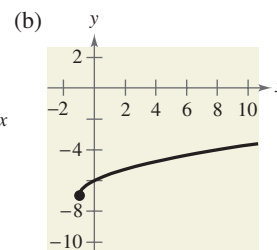
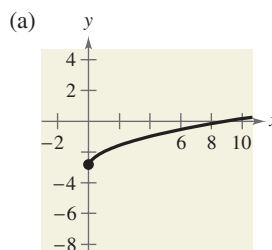
10. Use the graph of $f(x) = x^3$ to write an equation for each function whose graph is shown.



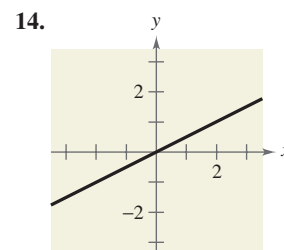
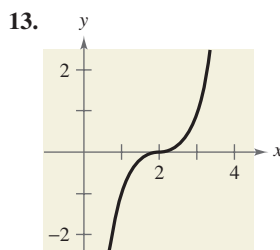
11. Use the graph of $f(x) = |x|$ to write an equation for each function whose graph is shown.

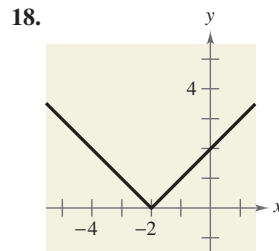
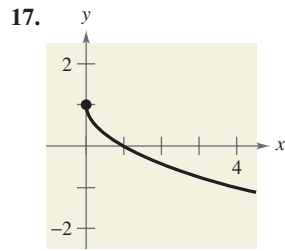
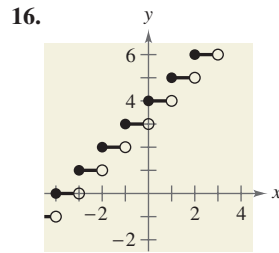
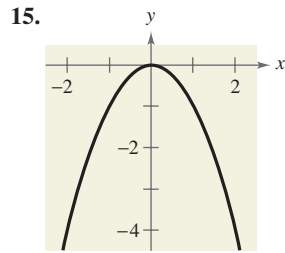


12. Use the graph of $f(x) = \sqrt{x}$ to write an equation for each function whose graph is shown.



In Exercises 13–18, identify the parent function and the transformation shown in the graph. Write an equation for the function shown in the graph.





In Exercises 19–42, g is related to one of the parent functions described in this chapter. (a) Identify the parent function f . (b) Describe the sequence of transformations from f to g . (c) Sketch the graph of g . (d) Use function notation to write g in terms of f .

- | | |
|--------------------------------------|-------------------------------------|
| 19. $g(x) = 12 - x^2$ | 20. $g(x) = (x - 8)^2$ |
| 21. $g(x) = x^3 + 7$ | 22. $g(x) = -x^3 - 1$ |
| 23. $g(x) = \frac{2}{3}x^2 + 4$ | 24. $g(x) = 2(x - 7)^2$ |
| 25. $g(x) = 2 - (x + 5)^2$ | 26. $g(x) = -(x + 10)^2 + 5$ |
| 27. $g(x) = \sqrt{3x}$ | 28. $g(x) = \sqrt{\frac{1}{4}x}$ |
| 29. $g(x) = (x - 1)^3 + 2$ | 30. $g(x) = (x + 3)^3 - 10$ |
| 31. $g(x) = - x - 2$ | 32. $g(x) = 6 - x + 5 $ |
| 33. $g(x) = - x + 4 + 8$ | 34. $g(x) = -x + 3 + 9$ |
| 35. $g(x) = 3 - \lfloor x \rfloor$ | 36. $g(x) = 2\lfloor x + 5 \rfloor$ |
| 37. $g(x) = \sqrt{x - 9}$ | 38. $g(x) = \sqrt{x + 4} + 8$ |
| 39. $g(x) = \sqrt{7 - x} - 2$ | 40. $g(x) = -\sqrt{x + 1} - 6$ |
| 41. $g(x) = \sqrt{\frac{1}{2}x} - 4$ | 42. $g(x) = \sqrt{3x} + 1$ |

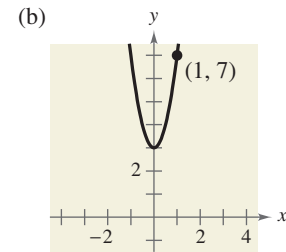
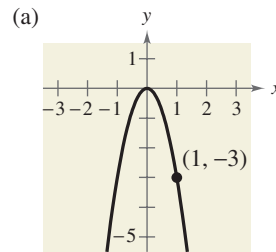
In Exercises 43–50, write an equation for the function that is described by the given characteristics.

43. The shape of $f(x) = x^2$, but moved two units to the right and eight units downward
44. The shape of $f(x) = x^2$, but moved three units to the left, seven units upward, and reflected in the x -axis
45. The shape of $f(x) = x^3$, but moved 13 units to the right
46. The shape of $f(x) = x^3$, but moved six units to the left, six units downward, and reflected in the y -axis
47. The shape of $f(x) = |x|$, but moved 10 units upward and reflected in the x -axis
48. The shape of $f(x) = |x|$, but moved one unit to the left and seven units downward

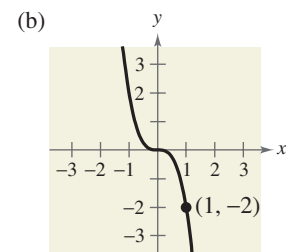
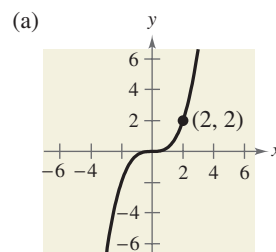
49. The shape of $f(x) = \sqrt{x}$, but moved six units to the left and reflected in both the x -axis and the y -axis

50. The shape of $f(x) = \sqrt{x}$, but moved nine units downward and reflected in both the x -axis and the y -axis

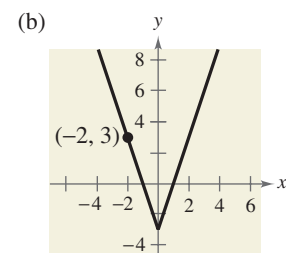
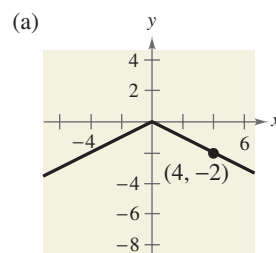
51. Use the graph of $f(x) = x^2$ to write an equation for each function whose graph is shown.



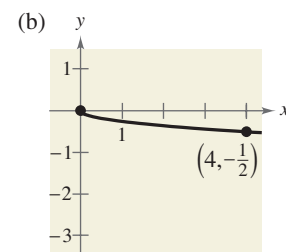
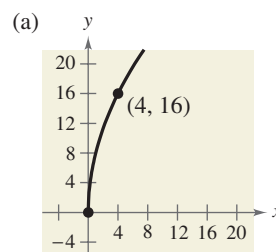
52. Use the graph of $f(x) = x^3$ to write an equation for each function whose graph is shown.



53. Use the graph of $f(x) = |x|$ to write an equation for each function whose graph is shown.



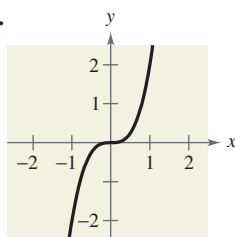
54. Use the graph of $f(x) = \sqrt{x}$ to write an equation for each function whose graph is shown.



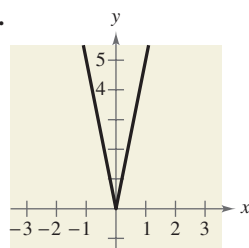
82 Chapter 1 Functions and Their Graphs

In Exercises 55–60, identify the parent function and the transformation shown in the graph. Write an equation for the function shown in the graph. Then use a graphing utility to verify your answer.

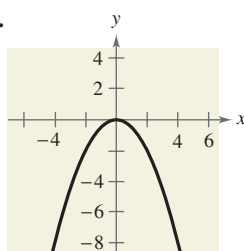
55.



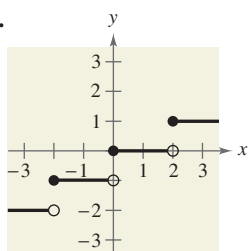
56.



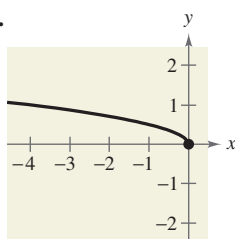
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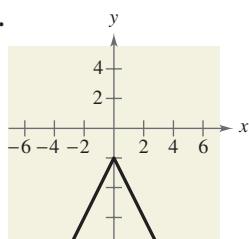
58.



59.

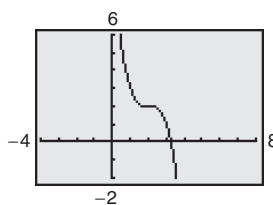


60.

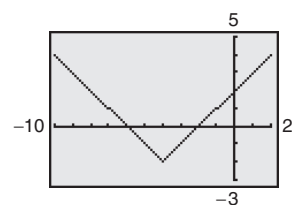


Graphical Analysis In Exercises 61–64, use the viewing window shown to write a possible equation for the transformation of the parent function.

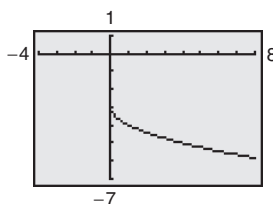
61.



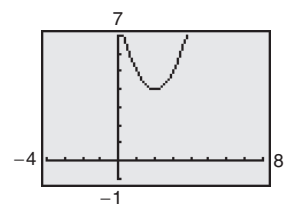
62.



63.

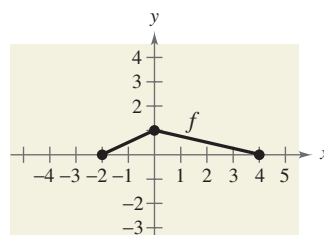


64.



Graphical Reasoning In Exercises 65 and 66, use the graph of f to sketch the graph of g . To print an enlarged copy of the graph, go to the website www.mathgraphs.com.

65.



(a) $g(x) = f(x) + 2$

(b) $g(x) = f(x) - 1$

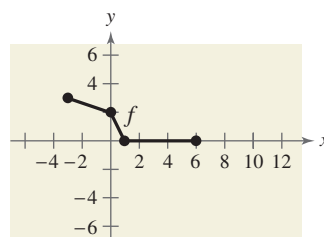
(c) $g(x) = f(-x)$

(d) $g(x) = -2f(x)$

(e) $g(x) = f(4x)$

(f) $g(x) = f(\frac{1}{2}x)$

66.



(a) $g(x) = f(x) - 5$

(b) $g(x) = f(x) + \frac{1}{2}$

(c) $g(x) = f(-x)$

(d) $g(x) = -4f(x)$

(e) $g(x) = f(2x) + 1$

(f) $g(x) = f(\frac{1}{4}x) - 2$

Model It

67. Fuel Use The amounts of fuel F (in billions of gallons) used by trucks from 1980 through 2002 can be approximated by the function

$$F = f(t) = 20.6 + 0.035t^2, \quad 0 \leq t \leq 22$$

where t represents the year, with $t = 0$ corresponding to 1980. (Source: U.S. Federal Highway Administration)

(a) Describe the transformation of the parent function $f(x) = x^2$. Then sketch the graph over the specified domain.

(b) Find the average rate of change of the function from 1980 to 2002. Interpret your answer in the context of the problem.

(c) Rewrite the function so that $t = 0$ represents 1990. Explain how you got your answer.

(d) Use the model from part (c) to predict the amount of fuel used by trucks in 2010. Does your answer seem reasonable? Explain.

- 68. Finance** The amounts M (in trillions of dollars) of mortgage debt outstanding in the United States from 1990 through 2002 can be approximated by the function

$$M = f(t) = 0.0054(t + 20.396)^2, \quad 0 \leq t \leq 12$$

where t represents the year, with $t = 0$ corresponding to 1990. (Source: Board of Governors of the Federal Reserve System)

- Describe the transformation of the parent function $f(x) = x^2$. Then sketch the graph over the specified domain.
- Rewrite the function so that $t = 0$ represents 2000. Explain how you got your answer.

Synthesis

True or False? In Exercises 69 and 70, determine whether the statement is true or false. Justify your answer.

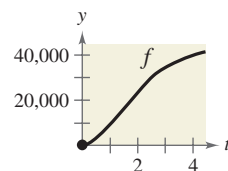
- 69.** The graphs of

$$f(x) = |x| + 6 \quad \text{and} \quad f(x) = |-x| + 6$$

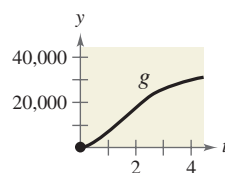
are identical.

- 70.** If the graph of the parent function $f(x) = x^2$ is moved six units to the right, three units upward, and reflected in the x -axis, then the point $(-2, 19)$ will lie on the graph of the transformation.

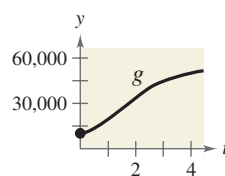
- 71. Describing Profits** Management originally predicted that the profits from the sales of a new product would be approximated by the graph of the function f shown. The actual profits are shown by the function g along with a verbal description. Use the concepts of transformations of graphs to write g in terms of f .



- (a) The profits were only three-fourths as large as expected.



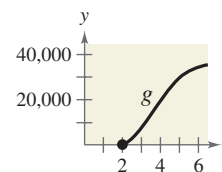
- (b) The profits were consistently \$10,000 greater than predicted.



Section 1.7 Transformations of Functions

83

- (c) There was a two-year delay in the introduction of the product. After sales began, profits grew as expected.



- Explain why the graph of $y = -f(x)$ is a reflection of the graph of $y = f(x)$ about the x -axis.
- The graph of $y = f(x)$ passes through the points $(0, 1)$, $(1, 2)$, and $(2, 3)$. Find the corresponding points on the graph of $y = f(x + 2) - 1$.
- Think About It** You can use either of two methods to graph a function: plotting points or translating a parent function as shown in this section. Which method of graphing do you prefer to use for each function? Explain.
 - $f(x) = 3x^2 - 4x + 1$
 - $f(x) = 2(x - 1)^2 - 6$

Skills Review

In Exercises 75–82, perform the operation and simplify.

75. $\frac{4}{x} + \frac{4}{1-x}$

76. $\frac{2}{x+5} - \frac{2}{x-5}$

77. $\frac{3}{x-1} - \frac{2}{x(x-1)}$

78. $\frac{x}{x-5} + \frac{1}{2}$

79. $(x-4)\left(\frac{1}{\sqrt{x^2-4}}\right)$

80. $\left(\frac{x}{x^2-4}\right)\left(\frac{x^2-x-2}{x^2}\right)$

81. $(x^2-9) \div \left(\frac{x+3}{5}\right)$

82. $\left(\frac{x}{x^2-3x-28}\right) \div \left(\frac{x^2+3x}{x^2+5x+4}\right)$

In Exercises 83 and 84, evaluate the function at the specified values of the independent variable and simplify.

83. $f(x) = x^2 - 6x + 11$

(a) $f(-3)$ (b) $f(-\frac{1}{2})$ (c) $f(x-3)$

84. $f(x) = \sqrt{x+10} - 3$

(a) $f(-10)$ (b) $f(26)$ (c) $f(x-10)$

In Exercises 85–88, find the domain of the function.

85. $f(x) = \frac{2}{11-x}$

86. $f(x) = \frac{\sqrt{x-3}}{x-8}$

87. $f(x) = \sqrt{81-x^2}$

88. $f(x) = \sqrt[3]{4-x^2}$