1.1 Rectangular Coordinates

What you should learn

- Plot points in the Cartesian plane.
- Use the Distance Formula to find the distance between two points.
- Use the Midpoint Formula to find the midpoint of a line segment.
- Use a coordinate plane and geometric formulas to model and solve real-life problems.

Why you should learn it

The Cartesian plane can be used to represent relationships between two variables. For instance, in Exercise 60 on page 12, a graph represents the minimum wage in the United States from 1950 to 2004.

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The Cartesian Plane

Just as you can represent real numbers by points on a real number line, you can represent ordered pairs of real numbers by points in a plane called the **rectangular coordinate system,** or the **Cartesian plane,** named after the French mathematician René Descartes (1596–1650).

The Cartesian plane is formed by using two real number lines intersecting at right angles, as shown in Figure 1.1. The horizontal real number line is usually called the ***x*-axis,** and the vertical real number line is usually called the ***y*-axis.** The point of intersection of these two axes is the **origin,** and the two axes divide the plane into four parts called **quadrants.**

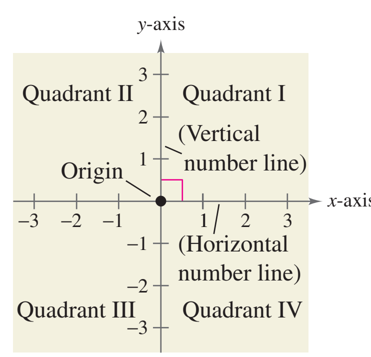


FIGURE 1.1

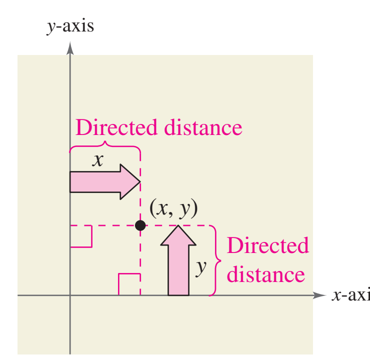


FIGURE 1.2

Each point in the plane corresponds to an **ordered pair** (x, y) of real numbers x and y, called **coordinates** of the point. The ***x*-coordinate** represents the directed distance from the y-axis to the point, and the ***y*-coordinate** represents the directed distance from the x-axis to the point, as shown in Figure 1.2.

Directed distance from y-axis $\quad (x, y) \quad$ Directed distance from x-axis

The notation (x, y) denotes both a point in the plane and an open interval on the real number line. The context will tell you which meaning is intended.

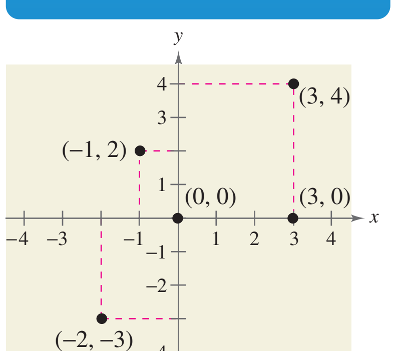


FIGURE 1.3

Example 1 Plotting Points in the Cartesian Plane

Plot the points $(-1, 2)$, $(3, 4)$, $(0, 0)$, $(3, 0)$, and $(-2, -3)$.

Solution

To plot the point $(-1, 2)$, imagine a vertical line through -1 on the x-axis and a horizontal line through 2 on the y-axis. The intersection of these two lines is the point $(-1, 2)$. The other four points can be plotted in a similar way, as shown in Figure 1.3.

CHECKPOINT Now try Exercise 3.

The beauty of a rectangular coordinate system is that it allows you to *see* relationships between two variables. It would be difficult to overestimate the importance of Descartes's introduction of coordinates in the plane. Today, his ideas are in common use in virtually every scientific and business-related field.

Example 2 Sketching a Scatter Plot

Year, t	Amount, A
1990	475
1991	577
1992	521
1993	569
1994	609
1995	562
1996	707
1997	723
1998	718
1999	648
2000	495
2001	476
2002	527
2003	464

From 1990 through 2003, the amounts A (in millions of dollars) spent on skiing equipment in the United States are shown in the table, where t represents the year. Sketch a scatter plot of the data. (Source: National Sporting Goods Association)

Solution

To sketch a *scatter plot* of the data shown in the table, you simply represent each pair of values by an ordered pair (t, A) and plot the resulting points, as shown in Figure 1.4. For instance, the first pair of values is represented by the ordered pair (1990, 475). Note that the break in the t-axis indicates that the numbers between 0 and 1990 have been omitted.

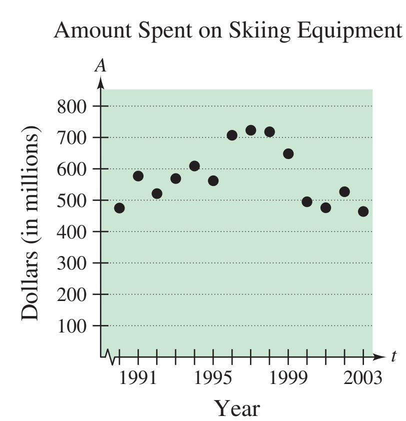


FIGURE 1.4

CHECKPOINT Now try Exercise 21.

In Example 2, you could have let $t = 1$ represent the year 1990. In that case, the horizontal axis would not have been broken, and the tick marks would have been labeled 1 through 14 (instead of 1990 through 2003).

Technology

The scatter plot in Example 2 is only one way to represent the data graphically. You could also represent the data using a bar graph and a line graph. If you have access to a graphing utility, try using it to represent graphically the data given in Example 2.

The *HM mathSpace*® CD-ROM and *Eduspace*® for this text contain additional resources related to the concepts discussed in this chapter.

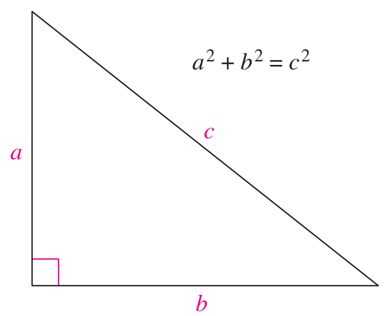


FIGURE 1.5

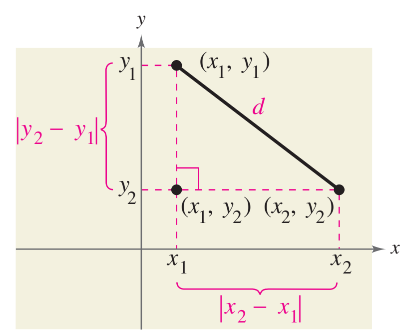


FIGURE 1.6

The Pythagorean Theorem and the Distance Formula

The following famous theorem is used extensively throughout this course.

Pythagorean Theorem

For a right triangle with hypotenuse of length c and sides of lengths a and b, you have $a^2 + b^2 = c^2$, as shown in Figure 1.5. (The converse is also true. That is, if $a^2 + b^2 = c^2$, then the triangle is a right triangle.)

Suppose you want to determine the distance d between two points (x_1, y_1) and (x_2, y_2) in the plane. With these two points, a right triangle can be formed, as shown in Figure 1.6. The length of the vertical side of the triangle is $|y_2 - y_1|$, and the length of the horizontal side is $|x_2 - x_1|$. By the Pythagorean Theorem, you can write

$$d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$$

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

This result is the **Distance Formula.**

The Distance Formula

The distance d between the points (x_1, y_1) and (x_2, y_2) in the plane is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Example 3 Finding a Distance

Find the distance between the points $(-2, 1)$ and $(3, 4)$.

Algebraic Solution

Let $(x_1, y_1) = (-2, 1)$ and $(x_2, y_2) = (3, 4)$. Then apply the Distance Formula.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{[3 - (-2)]^2 + (4 - 1)^2} && \text{Substitute for } x_1, y_1, x_2, \text{ and } y_2. \\ &= \sqrt{(5)^2 + (3)^2} && \text{Simplify.} \\ &= \sqrt{34} && \text{Simplify.} \\ &\approx 5.83 && \text{Use a calculator.} \end{aligned}$$

So, the distance between the points is about 5.83 units. You can use the Pythagorean Theorem to check that the distance is correct.

$$\begin{aligned} d^2 &\stackrel{?}{=} 3^2 + 5^2 && \text{Pythagorean Theorem} \\ \left(\sqrt{34}\right)^2 &\stackrel{?}{=} 3^2 + 5^2 && \text{Substitute for } d. \\ 34 &= 34 && \text{Distance checks. } \checkmark \end{aligned}$$

CHECKPOINT Now try Exercises 31(a) and (b).

Graphical Solution

Use centimeter graph paper to plot the points $A(-2, 1)$ and $B(3, 4)$. Carefully sketch the line segment from A to B. Then use a centimeter ruler to measure the length of the segment.

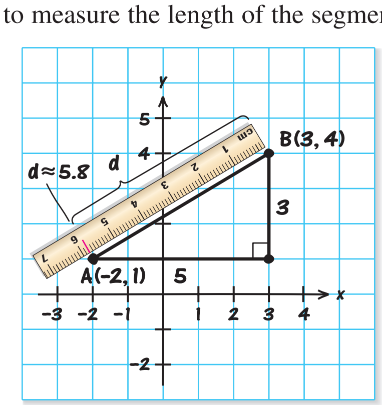


FIGURE 1.7

The line segment measures about 5.8 centimeters, as shown in Figure 1.7. So, the distance between the points is about 5.8 units.

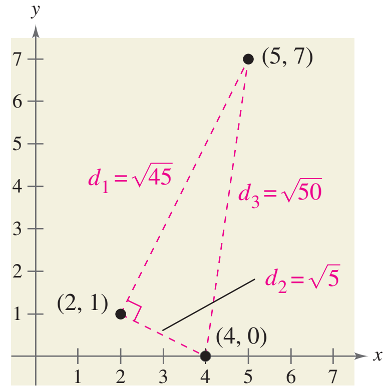


FIGURE 1.8

Example 4 **Verifying a Right Triangle**

Show that the points (2, 1), (4, 0), and (5, 7) are vertices of a right triangle.

Solution

The three points are plotted in Figure 1.8. Using the Distance Formula, you can find the lengths of the three sides as follows.

$$d_1 = \sqrt{(5-2)^2 + (7-1)^2} = \sqrt{9+36} = \sqrt{45}$$

$$d_2 = \sqrt{(4-2)^2 + (0-1)^2} = \sqrt{4+1} = \sqrt{5}$$

$$d_3 = \sqrt{(5-4)^2 + (7-0)^2} = \sqrt{1+49} = \sqrt{50}$$

Because

$$(d_1)^2 + (d_2)^2 = 45 + 5 = 50 = (d_3)^2$$

you can conclude by the Pythagorean Theorem that the triangle must be a right triangle.

CHECKPOINT Now try Exercise 41.

An overhead projector is useful for showing how to plot points and equations. Try projecting a grid onto the chalkboard and then plotting points on the chalkboard, or try using overhead markers and graph directly on the transparency. A viewscreen, a device used with an overhead projector to project a graphing calculator's screen image, is also useful.

The Midpoint Formula

To find the **midpoint** of the line segment that joins two points in a coordinate plane, you can simply find the average values of the respective coordinates of the two endpoints using the **Midpoint Formula.**

Exercises 43–46 on page 10 help develop a general understanding of the Midpoint Formula.

The Midpoint Formula

The midpoint of the line segment joining the points (x_1, y_1) and (x_2, y_2) is given by the Midpoint Formula

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

For a proof of the Midpoint Formula, see Proofs in Mathematics on page 124.

Example 5 **Finding a Line Segment's Midpoint**

Find the midpoint of the line segment joining the points $(-5, -3)$ and $(9, 3)$.

Solution

Let $(x_1, y_1) = (-5, -3)$ and $(x_2, y_2) = (9, 3)$.

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \qquad \text{Midpoint Formula}$$

$$= \left(\frac{-5 + 9}{2}, \frac{-3 + 3}{2}\right) \qquad \text{Substitute for } x_1, y_1, x_2, \text{ and } y_2.$$

$$= (2, 0) \qquad \text{Simplify.}$$

The midpoint of the line segment is (2, 0), as shown in Figure 1.9.

CHECKPOINT Now try Exercise 31(c).

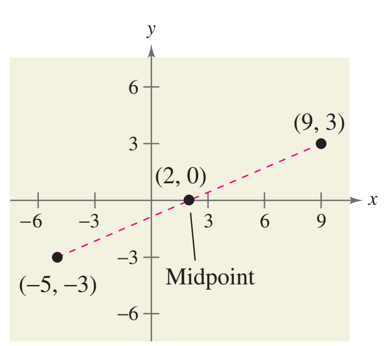


FIGURE 1.9

Applications

Example 6 Finding the Length of a Pass

During the third quarter of the 2004 Sugar Bowl, the quarterback for Louisiana State University threw a pass from the 28-yard line, 40 yards from the sideline. The pass was caught by a wide receiver on the 5-yard line, 20 yards from the same sideline, as shown in Figure 1.10. How long was the pass?

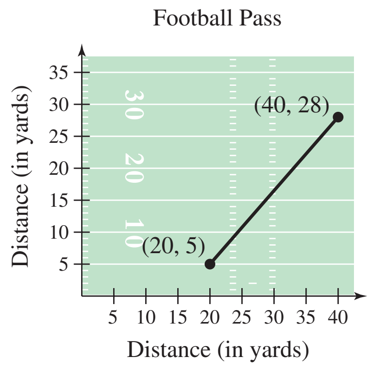


FIGURE 1.10

Solution

You can find the length of the pass by finding the distance between the points (40, 28) and (20, 5).

$$
\begin{aligned}
d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula}\\
&= \sqrt{(40 - 20)^2 + (28 - 5)^2} && \text{Substitute for } x_1, y_1, x_2, \text{ and } y_2.\\
&= \sqrt{400 + 529} && \text{Simplify.}\\
&= \sqrt{929} && \text{Simplify.}\\
&\approx 30 && \text{Use a calculator.}
\end{aligned}
$$

So, the pass was about 30 yards long.

CHECKPOINT Now try Exercise 47.

In Example 6, the scale along the goal line does not normally appear on a football field. However, when you use coordinate geometry to solve real-life problems, you are free to place the coordinate system in any way that is convenient for the solution of the problem.

Example 7 Estimating Annual Revenue

FedEx Corporation had annual revenues of \$20.6 billion in 2002 and \$24.7 billion in 2004. Without knowing any additional information, what would you estimate the 2003 revenue to have been? (Source: FedEx Corp.)

Solution

One solution to the problem is to assume that revenue followed a linear pattern. With this assumption, you can estimate the 2003 revenue by finding the midpoint of the line segment connecting the points (2002, 20.6) and (2004, 24.7).

$$
\begin{aligned}
\text{Midpoint} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) && \text{Midpoint Formula}\\
&= \left(\frac{2002 + 2004}{2}, \frac{20.6 + 24.7}{2}\right) && \text{Substitute for } x_1, y_1, x_2, \text{ and } y_2.\\
&= (2003, 22.65) && \text{Simplify.}
\end{aligned}
$$

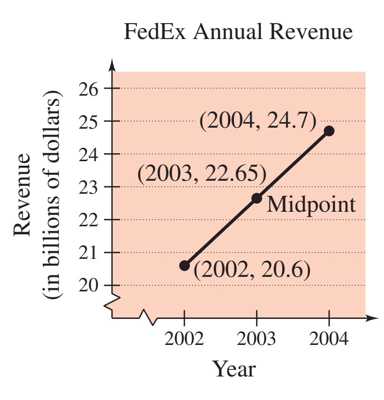
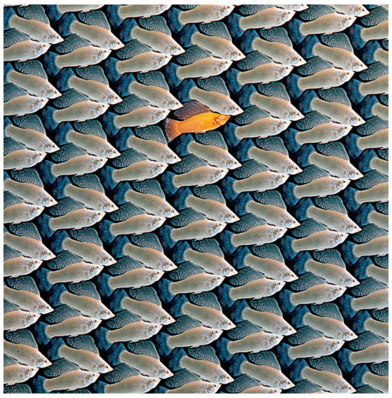


FIGURE 1.11

So, you would estimate the 2003 revenue to have been about \$22.65 billion, as shown in Figure 1.11. (The actual 2003 revenue was \$22.5 billion.)

CHECKPOINT Now try Exercise 49.



Paul Morrell

Much of computer graphics, including this computer-generated goldfish tessellation, consists of transformations of points in a coordinate plane. One type of transformation, a translation, is illustrated in Example 8. Other types include reflections, rotations, and stretches.

Activities

1. Set up a Cartesian plane and plot the points $(3, 0)$ and $(-4, 1)$.
2. Find the distance between $(3, 5)$ and $(-1, 2)$.
   Answer: 5
3. Find the midpoint of the line segment joining the points $(-1, -4)$ and $(3, -2)$.
   Answer: $(1, -3)$

Example 8 Translating Points in the Plane

The triangle in Figure 1.12 has vertices at the points $(-1, 2)$, $(1, -4)$, and $(2, 3)$. Shift the triangle three units to the right and two units upward and find the vertices of the shifted triangle, as shown in Figure 1.13.

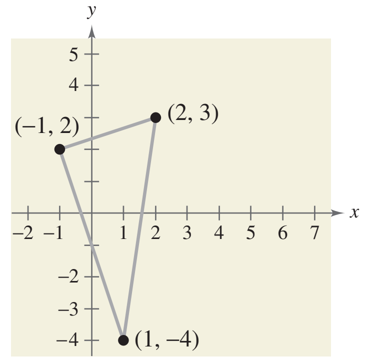


FIGURE 1.12

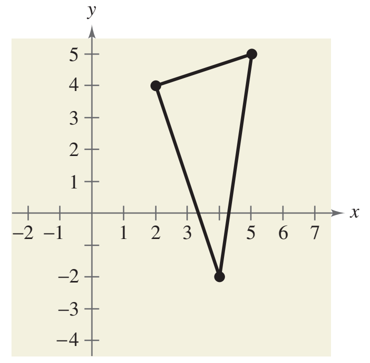


FIGURE 1.13

Solution

To shift the vertices three units to the right, add 3 to each of the x-coordinates. To shift the vertices two units upward, add 2 to each of the y-coordinates.

Original Point	*Translated Point*
$(-1, 2)$	$(-1 + 3, 2 + 2) = (2, 4)$
$(1, -4)$	$(1 + 3, -4 + 2) = (4, -2)$
$(2, 3)$	$(2 + 3, 3 + 2) = (5, 5)$

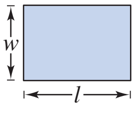
CHECKPOINT Now try Exercise 51.

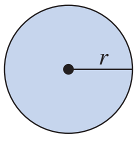
The figures provided with Example 8 were not really essential to the solution. Nevertheless, it is strongly recommended that you develop the habit of including sketches with your solutions—even if they are not required.

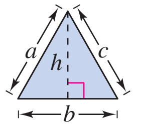
The following geometric formulas are used at various times throughout this course. For your convenience, these formulas along with several others are also provided on the inside back cover of this text.

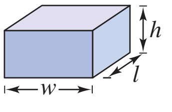
Common Formulas for Area A, Perimeter P, Circumference C, and Volume V

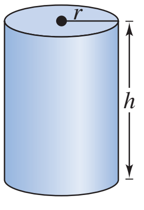
Rectangle	*Circle*	*Triangle*	*Rectangular Solid*	*Circular Cylinder*	*Sphere*
$A = lw$	$A = \pi r^2$	$A = \frac{1}{2}bh$	$V = lwh$	$V = \pi r^2 h$	$V = \frac{4}{3}\pi r^3$
$P = 2l + 2w$	$C = 2\pi r$	$P = a + b + c$			

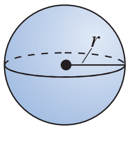












Example 9 Using a Geometric Formula

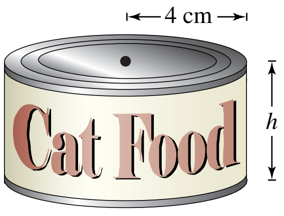


FIGURE 1.14

A cylindrical can has a volume of 200 cubic centimeters (cm^3) and a radius of 4 centimeters (cm), as shown in Figure 1.14. Find the height of the can.

Solution

The formula for the *volume of a cylinder* is $V = \pi r^2 h$. To find the height of the can, solve for h.

$$h = \frac{V}{\pi r^2}$$

Then, using $V = 200$ and $r = 4$, find the height.

$$h = \frac{200}{\pi(4)^2} \qquad \text{Substitute 200 for } V \text{ and 4 for } r.$$

$$= \frac{200}{16\pi} \qquad \text{Simplify denominator.}$$

$$\approx 3.98 \qquad \text{Use a calculator.}$$

Because the value of h was rounded in the solution, a check of the solution will not result in an equality. If the solution is valid, the expressions on each side of the equal sign will be approximately equal to each other.

$$V = \pi r^2 h \qquad \text{Write original equation.}$$

$$200 \stackrel{?}{\approx} \pi(4)^2(3.98) \qquad \text{Substitute 200 for } V\text{, 4 for } r\text{, and 3.98 for } h.$$

$$200 \approx 200.06 \qquad \text{Solution checks. } \checkmark$$

You can also use unit analysis to check that your answer is reasonable.

$$\frac{200\ \cancel{cm^3}}{16\pi\ \cancel{cm^2}} \approx 3.98 \text{ cm}$$

CHECKPOINT Now try Exercise 63.

Alternative Writing About Mathematics

Use your school's library, the Internet, or some other reference source to locate a set of real data that can be considered as ordered pairs. Decide which variables should be the x- and y-variables, and explain why you made this decision. List each ordered pair. Sketch a scatter plot of the data, choosing appropriate axes and scale. Be sure to label each axis and give the units for each variable if necessary. Describe any apparent trends in the data. What can you learn from your graph?

Writing About Mathematics

Extending the Example Example 8 shows how to translate points in a coordinate plane. Write a short paragraph describing how each of the following transformed points is related to the original point.

Original Point	*Transformed Point*
(x, y)	$(-x, y)$
(x, y)	$(x, -y)$
(x, y)	$(-x, -y)$

1.1 Exercises

The *HM mathSpace*® CD-ROM and *Eduspace*® for this text contain step-by-step solutions to all odd-numbered exercises. They also provide Tutorial Exercises for additional help.

VOCABULARY CHECK

1. Match each term with its definition.

(a) x-axis	(i) point of intersection of vertical axis and horizontal axis
(b) y-axis	(ii) directed distance from the x-axis
(c) origin	(iii) directed distance from the y-axis
(d) quadrants	(iv) four regions of the coordinate plane
(e) x-coordinate	(v) horizontal real number line
(f) y-coordinate	(vi) vertical real number line

In Exercises 2–4, fill in the blanks.

2. An ordered pair of real numbers can be represented in a plane called the rectangular coordinate system or the ________ plane.

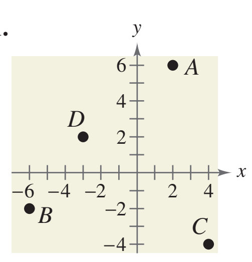
3. The ________ ________ is a result derived from the Pythagorean Theorem.

4. Finding the average values of the representative coordinates of the two endpoints of a line segment in a coordinate plane is also known as using the ________ ________.

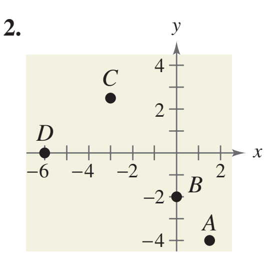
PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

In Exercises 1 and 2, approximate the coordinates of the points.

1.



2.



In Exercises 3–6, plot the points in the Cartesian plane.

3. $(-4, 2), (-3, -6), (0, 5), (1, -4)$

4. $(0, 0), (3, 1), (-2, 4), (1, -1)$

5. $(3, 8), (0.5, -1), (5, -6), (-2, 2.5)$

6. $\left(1, -\frac{1}{3}\right), \left(\frac{3}{4}, 3\right), (-3, 4), \left(-\frac{4}{3}, -\frac{3}{2}\right)$

In Exercises 7–10, find the coordinates of the point.

7. The point is located three units to the left of the y-axis and four units above the x-axis.

8. The point is located eight units below the x-axis and four units to the right of the y-axis.

9. The point is located five units below the x-axis and the coordinates of the point are equal.

10. The point is on the x-axis and 12 units to the left of the y-axis.

In Exercises 11–20, determine the quadrant(s) in which (x, y) is located so that the condition(s) is (are) satisfied.

11. $x > 0$ and $y < 0$

12. $x < 0$ and $y < 0$

13. $x = -4$ and $y > 0$

14. $x > 2$ and $y = 3$

15. $y < -5$

16. $x > 4$

17. $x < 0$ and $-y > 0$

18. $-x > 0$ and $y < 0$

19. $xy > 0$

20. $xy < 0$

In Exercises 21 and 22, sketch a scatter plot of the data shown in the table.

21. ***Number of Stores*** The table shows the number y of Wal-Mart stores for each year x from 1996 through 2003. (Source: Wal-Mart Stores, Inc.)

Year, x	Number of stores, y
1996	3054
1997	3406
1998	3599
1999	3985
2000	4189
2001	4414
2002	4688
2003	4906

22. ***Meteorology*** The table shows the lowest temperature on record y (in degrees Fahrenheit) in Duluth, Minnesota for each month x, where $x = 1$ represents January. (Source: NOAA)

Month, x	Temperature, y
1	−39
2	−39
3	−29
4	−5
5	17
6	27
7	35
8	32
9	22
10	8
11	−23
12	−34

In Exercises 23–26, find the distance between the points. (*Note:* In each case, the two points lie on the same horizontal or vertical line.)

23. $(6, -3), (6, 5)$

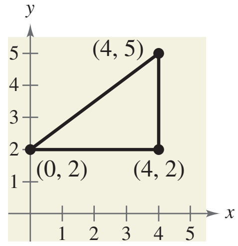
24. $(1, 4), (8, 4)$

25. $(-3, -1), (2, -1)$

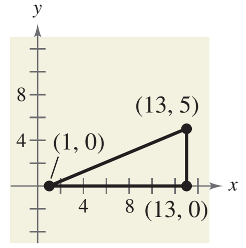
26. $(-3, -4), (-3, 6)$

In Exercises 27–30, (a) find the length of each side of the right triangle, and (b) show that these lengths satisfy the Pythagorean Theorem.

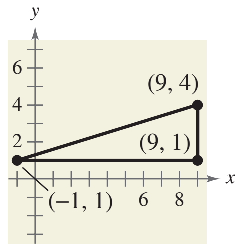
27.



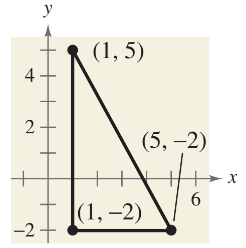
28.



29.



30.



In Exercises 31–40, (a) plot the points, (b) find the distance between the points, and (c) find the midpoint of the line segment joining the points.

31. $(1, 1), (9, 7)$

32. $(1, 12), (6, 0)$

33. $(-4, 10), (4, -5)$

34. $(-7, -4), (2, 8)$

35. $(-1, 2), (5, 4)$

36. $(2, 10), (10, 2)$

37. $\left(\frac{1}{2}, 1\right), \left(-\frac{5}{2}, \frac{4}{3}\right)$

38. $\left(-\frac{1}{3}, -\frac{1}{3}\right), \left(-\frac{1}{6}, -\frac{1}{2}\right)$

39. $(6.2, 5.4), (-3.7, 1.8)$

40. $(-16.8, 12.3), (5.6, 4.9)$

In Exercises 41 and 42, show that the points form the vertices of the indicated polygon.

41. Right triangle: $(4, 0), (2, 1), (-1, -5)$

42. Isosceles triangle: $(1, -3), (3, 2), (-2, 4)$

43. A line segment has (x_1, y_1) as one endpoint and (x_m, y_m) as its midpoint. Find the other endpoint (x_2, y_2) of the line segment in terms of x_1, y_1, x_m, and y_m.

44. Use the result of Exercise 43 to find the coordinates of the endpoint of a line segment if the coordinates of the other endpoint and midpoint are, respectively,

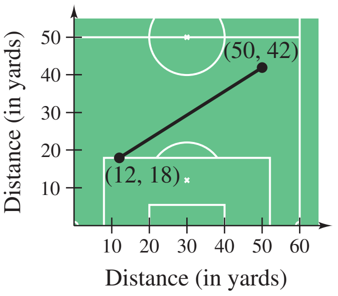
(a) $(1, -2), (4, -1)$ and (b) $(-5, 11), (2, 4)$.

45. Use the Midpoint Formula three times to find the three points that divide the line segment joining (x_1, y_1) and (x_2, y_2) into four parts.

46. Use the result of Exercise 45 to find the points that divide the line segment joining the given points into four equal parts.

(a) $(1, -2), (4, -1)$ (b) $(-2, -3), (0, 0)$

47. ***Sports*** A soccer player passes the ball from a point that is 18 yards from the endline and 12 yards from the sideline. The pass is received by a teammate who is 42 yards from the same endline and 50 yards from the same sideline, as shown in the figure. How long is the pass?



48. ***Flying Distance*** An airplane flies from Naples, Italy in a straight line to Rome, Italy, which is 120 kilometers north and 150 kilometers west of Naples. How far does the plane fly?

Sales **In Exercises 49 and 50, use the Midpoint Formula to estimate the sales of Big Lots, Inc. and Dollar Tree Stores, Inc. in 2002, given the sales in 2001 and 2003. Assume that the sales followed a linear pattern.** (Source: Big Lots, Inc.; Dollar Tree Stores, Inc.)

49. Big Lots

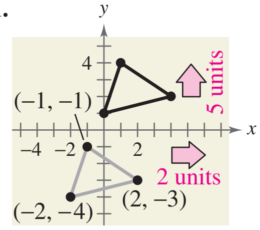
Year	Sales (in millions)
2001	\$3433
2003	\$4174

50. Dollar Tree

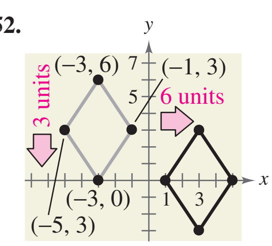
Year	Sales (in millions)
2001	\$1987
2003	\$2800

In Exercises 51–54, the polygon is shifted to a new position in the plane. Find the coordinates of the vertices of the polygon in its new position.

51.



52.



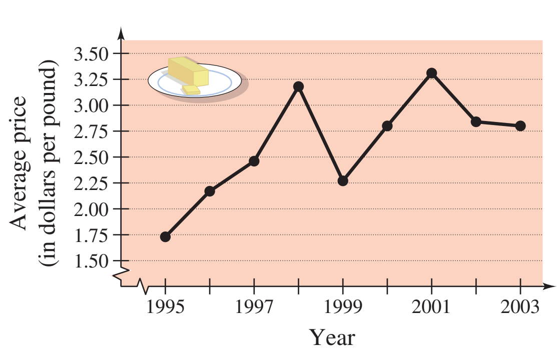
53. Original coordinates of vertices: $(-7, -2), (-2, 2),$ $(-2, -4), (-7, -4)$

Shift: eight units upward, four units to the right

54. Original coordinates of vertices: $(5, 8), (3, 6), (7, 6), (5, 2)$

Shift: 6 units downward, 10 units to the left

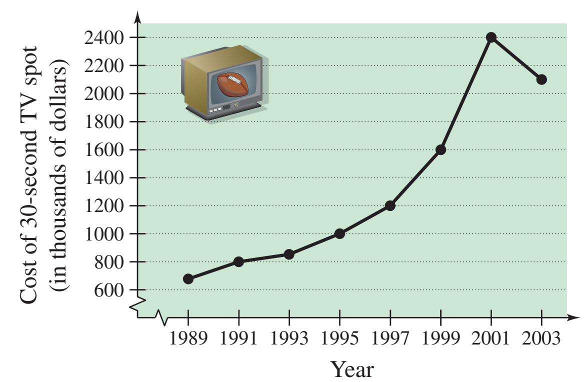
Retail Price **In Exercises 55 and 56, use the graph below, which shows the average retail price of 1 pound of butter from 1995 to 2003.** (Source: U.S. Bureau of Labor Statistics)



55. Approximate the highest price of a pound of butter shown in the graph. When did this occur?

56. Approximate the percent change in the price of butter from the price in 1995 to the highest price shown in the graph.

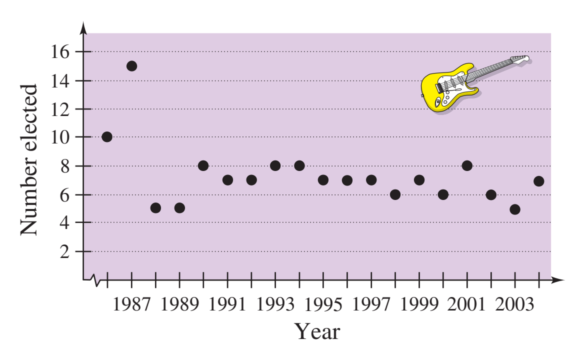
Advertising **In Exercises 57 and 58, use the graph below, which shows the cost of a 30-second television spot (in thousands of dollars) during the Super Bowl from 1989 to 2003.** (Source: USA Today Research and CNN)



57. Approximate the percent increase in the cost of a 30-second spot from Super Bowl XXIII in 1989 to Super Bowl XXXV in 2001.

58. Estimate the percent increase in the cost of a 30-second spot (a) from Super Bowl XXIII in 1989 to Super Bowl XXVII in 1993 and (b) from Super Bowl XXVII in 1993 to Super Bowl XXXVII in 2003.

59. ***Music*** The graph shows the numbers of recording artists who were elected to the Rock and Roll Hall of Fame from 1986 to 2004.

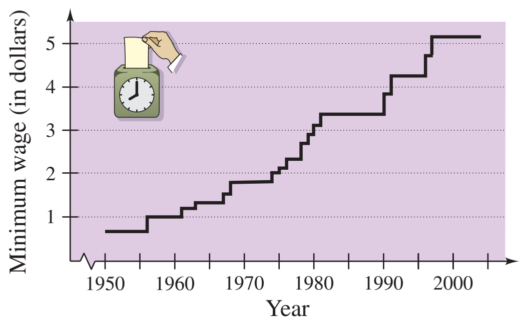


(a) Describe any trends in the data. From these trends, predict the number of artists elected in 2008.

(b) Why do you think the numbers elected in 1986 and 1987 were greater in other years?

Model It

60. ***Labor Force*** Use the graph below, which shows the minimum wage in the United States (in dollars) from 1950 to 2004. (Source: U.S. Department of Labor)



(a) Which decade shows the greatest increase in minimum wage?

(b) Approximate the percent increases in the minimum wage from 1990 to 1995 and from 1995 to 2004.

(c) Use the percent increase from 1995 to 2004 to predict the minimum wage in 2008.

(d) Do you believe that your prediction in part (c) is reasonable? Explain.

61. ***Sales*** The Coca-Cola Company had sales of \$18,546 million in 1996 and \$21,900 million in 2004. Use the Midpoint Formula to estimate the sales in 1998, 2000, and 2002. Assume that the sales followed a linear pattern. (Source: The Coca-Cola Company)

62. ***Data Analysis: Exam Scores*** The table shows the mathematics entrance test scores x and the final examination scores y in an algebra course for a sample of 10 students.

x	22	29	35	40	44
y	53	74	57	66	79

x	48	53	58	65	76
y	90	76	93	83	99

(a) Sketch a scatter plot of the data.

(b) Find the entrance exam score of any student with a final exam score in the 80s.

(c) Does a higher entrance exam score imply a higher final exam score? Explain.

63. ***Volume of a Billiard Ball*** A billiard ball has a volume of 5.96 cubic inches. Find the radius of a billiard ball.

64. ***Length of a Tank*** The diameter of a cylindrical propane gas tank is 4 feet. The total volume of the tank is 603.2 cubic feet. Find the length of the tank.

65. ***Geometry*** A "Slow Moving Vehicle" sign has the shape of an equilateral triangle. The sign has a perimeter of 129 centimeters. Find the length of each side of the sign. Find the area of the sign.

66. ***Geometry*** The radius of a traffic cone is 14 centimeters and the lateral surface of the cone is 1617 square centimeters. Find the height of the cone.

67. ***Dimensions of a Room*** A room is 1.5 times as long as it is wide, and its perimeter is 25 meters.

(a) Draw a diagram that represents the problem. Identify the length as l and the width as w.

(b) Write l in terms of w and write an equation for the perimeter in terms of w.

(c) Find the dimensions of the room.

68. ***Dimensions of a Container*** The width of a rectangular storage container is 1.25 times its height. The length of the container is 16 inches and the volume of the container is 2000 cubic inches.

(a) Draw a diagram that represents the problem. Label the height, width, and length accordingly.

(b) Write w in terms of h and write an equation for the volume in terms of h.

(c) Find the dimensions of the container.

69. ***Data Analysis: Mail*** The table shows the number y of pieces of mail handled (in billions) by the U.S. Postal Service for each year x from 1996 through 2003. (Source: U.S. Postal Service)

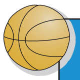
Year, x	Pieces of mail, y
1996	183
1997	191
1998	197
1999	202
2000	208
2001	207
2002	203
2003	202

(a) Sketch a scatter plot of the data.

(b) Approximate the year in which there was the greatest decrease in the number of pieces of mail handled.

(c) Why do you think the number of pieces of mail handled decreased?

70. ***Data Analysis: Athletics*** The table shows the numbers of men's M and women's W college basketball teams for each year x from 1994 through 2003. (Source: National Collegiate Athletic Association)



Year, x	Men's teams, M	Women's teams, W
1994	858	859
1995	868	864
1996	866	874
1997	865	879
1998	895	911
1999	926	940
2000	932	956
2001	937	958
2002	936	975
2003	967	1009

(a) Sketch scatter plots of these two sets of data on the same set of coordinate axes.

(b) Find the year in which the numbers of men's and women's teams were nearly equal.

(c) Find the year in which the difference between the numbers of men's and women's teams was the greatest. What was this difference?

71. ***Make a Conjecture*** Plot the points $(2, 1)$, $(-3, 5)$, and $(7, -3)$ on a rectangular coordinate system. Then change the sign of the x-coordinate of each point and plot the three new points on the same rectangular coordinate system. Make a conjecture about the location of a point when each of the following occurs.

(a) The sign of the x-coordinate is changed.

(b) The sign of the y-coordinate is changed.

(c) The signs of both the x- and y-coordinates are changed.

72. ***Collinear Points*** Three or more points are *collinear* if they all lie on the same line. Use the steps below to determine if the set of points $\{A(2, 3), B(2, 6), C(6, 3)\}$ and the set of points $\{A(8, 3), B(5, 2), C(2, 1)\}$ are collinear.

(a) For each set of points, use the Distance Formula to find the distances from A to B, from B to C, and from A to C. What relationship exists among these distances for each set of points?

(b) Plot each set of points in the Cartesian plane. Do all the points of either set appear to lie on the same line?

(c) Compare your conclusions from part (a) with the conclusions you made from the graphs in part (b). Make a general statement about how to use the Distance Formula to determine collinearity.

Synthesis

True or False? **In Exercises 73 and 74, determine whether the statement is true or false. Justify your answer.**

73. In order to divide a line segment into 16 equal parts, you would have to use the Midpoint Formula 16 times.

74. The points $(-8, 4)$, $(2, 11)$, and $(-5, 1)$ represent the vertices of an isosceles triangle.

75. ***Think About It*** When plotting points on the rectangular coordinate system, is it true that the scales on the x- and y-axes must be the same? Explain.

76. ***Proof*** Prove that the diagonals of the parallelogram in the figure intersect at their midpoints.

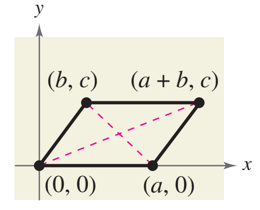


FIGURE FOR 76

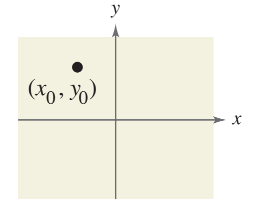
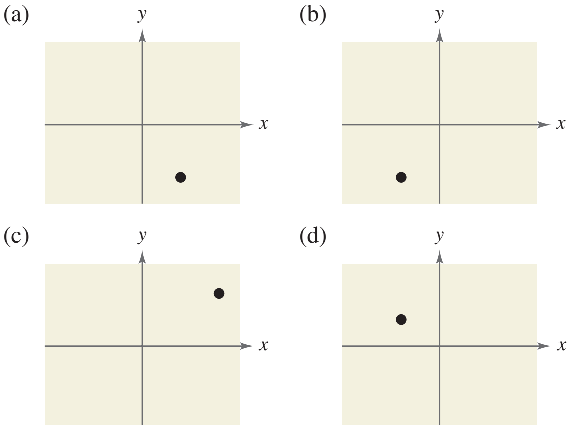


FIGURE FOR 77–80

In Exercises 77–80, use the plot of the point (x_0, y_0) in the figure. Match the transformation of the point with the correct plot. [The plots are labeled (a), (b), (c), and (d).]



77. $(x_0, -y_0)$

78. $(-2x_0, y_0)$

79. $\left(x_0, \frac{1}{2}y_0\right)$

80. $(-x_0, -y_0)$

Skills Review

In Exercises 81–88, solve the equation or inequality.

81. $2x + 1 = 7x - 4$

82. $\frac{1}{3}x + 2 = 5 - \frac{1}{6}x$

83. $x^2 - 4x - 7 = 0$

84. $2x^2 + 3x - 8 = 0$

85. $3x + 1 < 2(2 - x)$

86. $3x - 8 \geq \frac{1}{2}(10x + 7)$

87. $|x - 18| < 4$

88. $|2x + 15| \geq 11$