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A.3 Polynomials and Factoring

What you should learn

- Write polynomials in standard form.
- Add, subtract, and multiply polynomials.
- Use special products to multiply polynomials.
- Remove common factors from polynomials.
- Factor special polynomial forms.
- Factor trinomials as the product of two binomials.
- Factor polynomials by grouping.

Why you should learn it

Polynomials can be used to model and solve real-life problems. For instance, in Exercise 210 on page A34, a polynomial is used to model the stopping distance of an automobile.

Polynomials

The most common type of algebraic expression is the **polynomial.** Some examples are $2x + 5$, $3x^4 - 7x^2 + 2x + 4$, and $5x^2y^2 - xy + 3$. The first two are *polynomials in x* and the third is a *polynomial in x and y*. The terms of a polynomial in x have the form ax^k, where a is the **coefficient** and k is the **degree** of the term. For instance, the polynomial

$$2x^3 - 5x^2 + 1 = 2x^3 + (-5)x^2 + (0)x + 1$$

has coefficients 2, -5, 0, and 1.

Definition of a Polynomial in x

Let $a_0, a_1, a_2, \ldots, a_n$ be real numbers and let n be a nonnegative integer. A polynomial in x is an expression of the form

$$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

where $a_n \neq 0$. The polynomial is of **degree** n, a_n is the **leading coefficient,** and a_0 is the **constant term.**

Polynomials with one, two, and three terms are called **monomials, binomials,** and **trinomials,** respectively. In **standard form,** a polynomial is written with descending powers of x.

Example 1 Writing Polynomials in Standard Form

	Polynomial	*Standard Form*	*Degree*
a.	$4x^2 - 5x^7 - 2 + 3x$	$-5x^7 + 4x^2 + 3x - 2$	7
b.	$4 - 9x^2$	$-9x^2 + 4$	2
c.	8	$8 \quad (8 = 8x^0)$	0

CHECKPOINT Now try Exercise 11.

A polynomial that has all zero coefficients is called the zero polynomial, denoted by 0. No degree is assigned to this particular polynomial. For polynomials in more than one variable, the degree of a *term* is the sum of the exponents of the variables in the term. The degree of the *polynomial* is the highest degree of its terms. For instance, the degree of the polynomial $-2x^3y^6 + 4xy - x^7y^4$ is 11 because the sum of the exponents in the last term is the greatest. The leading coefficient of the polynomial is the coefficient of the highest-degree term. Expressions are not polynomials if a variable is underneath a radical or if a polynomial expression (with degree greater than 0) is in the denominator of a term. The following expressions are not polynomials.

$$x^3 - \sqrt{3x} = x^3 - (3x)^{1/2}$$ The exponent "1/2" is not an integer.

$$x^2 + \frac{5}{x} = x^2 + 5x^{-1}$$ The exponent "-1" is not a nonnegative integer.

Operations with Polynomials

You can add and subtract polynomials in much the same way you add and subtract real numbers. Simply add or subtract the *like terms* (terms having the same variables to the same powers) by adding their coefficients. For instance, $-3xy^2$ and $5xy^2$ are like terms and their sum is

$$\begin{aligned} -3xy^2 + 5xy^2 &= (-3 + 5)xy^2 \\ &= 2xy^2. \end{aligned}$$

STUDY TIP

When an expression inside parentheses is preceded by a negative sign, remember to distribute the negative sign to each term inside the parentheses, as shown.

$$\begin{aligned} -(x^2 - x + 3) \\ = -x^2 + x - 3 \end{aligned}$$

Example 2 Sums and Differences of Polynomials

a. $(5x^3 - 7x^2 - 3) + (x^3 + 2x^2 - x + 8)$

$= (5x^3 + x^3) + (-7x^2 + 2x^2) - x + (-3 + 8)$ Group like terms.

$= 6x^3 - 5x^2 - x + 5$ Combine like terms.

b. $(7x^4 - x^2 - 4x + 2) - (3x^4 - 4x^2 + 3x)$

$= 7x^4 - x^2 - 4x + 2 - 3x^4 + 4x^2 - 3x$ Distributive Property

$= (7x^4 - 3x^4) + (-x^2 + 4x^2) + (-4x - 3x) + 2$ Group like terms.

$= 4x^4 + 3x^2 - 7x + 2$ Combine like terms.

CHECKPOINT Now try Exercise 33.

To find the *product* of two polynomials, use the left and right Distributive Properties. For example, if you treat $5x + 7$ as a single quantity, you can multiply $3x - 2$ by $5x + 7$ as follows.

$$\begin{aligned} (3x - 2)(5x + 7) &= 3x(5x + 7) - 2(5x + 7) \\ &= (3x)(5x) + (3x)(7) - (2)(5x) - (2)(7) \\ &= 15x^2 + 21x - 10x - 14 \end{aligned}$$

Product of **F**irst terms | Product of **O**uter terms | Product of **I**nner terms | Product of **L**ast terms

$$= 15x^2 + 11x - 14$$

Note in this **FOIL Method** (which can only be used to multiply two binomials) that the outer (O) and inner (I) terms are like terms and can be combined.

Example 3 Finding a Product by the FOIL Method

Use the FOIL Method to find the product of $2x - 4$ and $x + 5$.

Solution

$$\begin{aligned} (2x - 4)(x + 5) &= \overset{\text{F}}{2x^2} + \overset{\text{O}}{10x} - \overset{\text{I}}{4x} - \overset{\text{L}}{20} \\ &= 2x^2 + 6x - 20 \end{aligned}$$

CHECKPOINT Now try Exercise 47.

Special Products

Some binomial products have special forms that occur frequently in algebra. You do not need to memorize these formulas because you can use the Distributive Property to multiply. However, becoming familiar with these formulas will enable you to manipulate the algebra more quickly.

Special Products

Let u and v be real numbers, variables, or algebraic expressions.

Special Product	*Example*
Sum and Difference of Same Terms	
$(u + v)(u - v) = u^2 - v^2$	$(x + 4)(x - 4) = x^2 - 4^2$ $= x^2 - 16$
Square of a Binomial	
$(u + v)^2 = u^2 + 2uv + v^2$	$(x + 3)^2 = x^2 + 2(x)(3) + 3^2$ $= x^2 + 6x + 9$
$(u - v)^2 = u^2 - 2uv + v^2$	$(3x - 2)^2 = (3x)^2 - 2(3x)(2) + 2^2$ $= 9x^2 - 12x + 4$
Cube of a Binomial	
$(u + v)^3 = u^3 + 3u^2v + 3uv^2 + v^3$	$(x + 2)^3 = x^3 + 3x^2(2) + 3x(2^2) + 2^3$ $= x^3 + 6x^2 + 12x + 8$
$(u - v)^3 = u^3 - 3u^2v + 3uv^2 - v^3$	$(x - 1)^3 = x^3 - 3x^2(1) + 3x(1^2) - 1^3$ $= x^3 - 3x^2 + 3x - 1$

Example 4 **Special Products**

Find each product.

a. $5x + 9$ and $5x - 9$ **b.** $x + y - 2$ and $x + y + 2$

Solution

a. The product of a sum and a difference of the *same* two terms has no middle term and takes the form $(u + v)(u - v) = u^2 - v^2$.

$$(5x + 9)(5x - 9) = (5x)^2 - 9^2 = 25x^2 - 81$$

b. By grouping $x + y$ in parentheses, you can write the product of the trinomials as a special product.

$$\begin{aligned}(x + y - 2)(x + y + 2) &= [\underset{\text{Difference}}{(x + y) - 2}][\underset{\text{Sum}}{(x + y) + 2}]\\ &= (x + y)^2 - 2^2 \qquad \text{Sum and difference of same terms}\\ &= x^2 + 2xy + y^2 - 4\end{aligned}$$

CHECKPOINT Now try Exercise 67.

Polynomials with Common Factors

The process of writing a polynomial as a product is called **factoring.** It is an important tool for solving equations and for simplifying rational expressions.

Unless noted otherwise, when you are asked to factor a polynomial, you can assume that you are looking for factors with integer coefficients. If a polynomial cannot be factored using integer coefficients, then it is **prime** or **irreducible over the integers.** For instance, the polynomial $x^2 - 3$ is irreducible over the integers. Over the *real numbers*, this polynomial can be factored as

$$x^2 - 3 = (x + \sqrt{3})(x - \sqrt{3}).$$

A polynomial is **completely factored** when each of its factors is prime. For instance

$$x^3 - x^2 + 4x - 4 = (x - 1)(x^2 + 4) \qquad \text{Completely factored}$$

is completely factored, but

$$x^3 - x^2 - 4x + 4 = (x - 1)(x^2 - 4) \qquad \text{Not completely factored}$$

is not completely factored. Its complete factorization is

$$x^3 - x^2 - 4x + 4 = (x - 1)(x + 2)(x - 2).$$

The simplest type of factoring involves a polynomial that can be written as the product of a monomial and another polynomial. The technique used here is the Distributive Property, $a(b + c) = ab + ac$, in the *reverse* direction.

$$ab + ac = a(b + c) \qquad a \text{ is a common factor.}$$

Removing (factoring out) any common factors is the first step in completely factoring a polynomial.

Example 5 Removing Common Factors

Factor each expression.

a. $6x^3 - 4x$

b. $-4x^2 + 12x - 16$

c. $(x - 2)(2x) + (x - 2)(3)$

Solution

a. $6x^3 - 4x = 2x(3x^2) - 2x(2)$ $\qquad$ $2x$ is a common factor.

$\qquad = 2x(3x^2 - 2)$

b. $-4x^2 + 12x - 16 = -4(x^2) + (-4)(-3x) + (-4)4$ $\qquad$ -4 is a common factor.

$\qquad = -4(x^2 - 3x + 4)$

c. $(x - 2)(2x) + (x - 2)(3) = (x - 2)(2x + 3)$ $\qquad$ $(x - 2)$ is a common factor.

CHECKPOINT Now try Exercise 91.

Factoring Special Polynomial Forms

Some polynomials have special forms that arise from the special product forms on page A25. You should learn to recognize these forms so that you can factor such polynomials easily.

Factoring Special Polynomial Forms

Factored Form	*Example*
Difference of Two Squares	
$u^2 - v^2 = (u + v)(u - v)$	$9x^2 - 4 = (3x)^2 - 2^2 = (3x + 2)(3x - 2)$
Perfect Square Trinomial	
$u^2 + 2uv + v^2 = (u + v)^2$	$x^2 + 6x + 9 = x^2 + 2(x)(3) + 3^2 = (x + 3)^2$
$u^2 - 2uv + v^2 = (u - v)^2$	$x^2 - 6x + 9 = x^2 - 2(x)(3) + 3^2 = (x - 3)^2$
Sum or Difference of Two Cubes	
$u^3 + v^3 = (u + v)(u^2 - uv + v^2)$	$x^3 + 8 = x^3 + 2^3 = (x + 2)(x^2 - 2x + 4)$
$u^3 - v^3 = (u - v)(u^2 + uv + v^2)$	$27x^3 - 1 = (3x)^3 - 1^3 = (3x - 1)(9x^2 + 3x + 1)$

One of the easiest special polynomial forms to factor is the difference of two squares. The factored form is always a set of *conjugate pairs*.

$$u^2 - v^2 = (u + v)(u - v)$$ Conjugate pairs

Difference Opposite signs

To recognize perfect square terms, look for coefficients that are squares of integers and variables raised to *even powers*.

> **STUDY TIP**
>
> In Example 6, note that the first step in factoring a polynomial is to check for any common factors. Once the common factors are removed, it is often possible to recognize patterns that were not immediately obvious.

Example 6 **Removing a Common Factor First**

$$
\begin{aligned}
3 - 12x^2 &= 3(1 - 4x^2) && \text{3 is a common factor.}\\
&= 3[1^2 - (2x)^2]\\
&= 3(1 + 2x)(1 - 2x) && \text{Difference of two squares}
\end{aligned}
$$

CHECKPOINT Now try Exercise 105.

Example 7 **Factoring the Difference of Two Squares**

a. $(x + 2)^2 - y^2 = [(x + 2) + y][(x + 2) - y]$

$= (x + 2 + y)(x + 2 - y)$

b.
$$
\begin{aligned}
16x^4 - 81 &= (4x^2)^2 - 9^2\\
&= (4x^2 + 9)(4x^2 - 9) && \text{Difference of two squares}\\
&= (4x^2 + 9)[(2x)^2 - 3^2]\\
&= (4x^2 + 9)(2x + 3)(2x - 3) && \text{Difference of two squares}
\end{aligned}
$$

CHECKPOINT Now try Exercise 109.

A **perfect square trinomial** is the square of a binomial, and it has the following form.

$$u^2 + 2uv + v^2 = (u + v)^2 \qquad \text{or} \qquad u^2 - 2uv + v^2 = (u - v)^2$$

Like signs Like signs

Note that the first and last terms are squares and the middle term is twice the product of u and v.

Example 8 **Factoring Perfect Square Trinomials**

Factor each trinomial.

a. $x^2 - 10x + 25$

b. $16x^2 + 24x + 9$

Solution

a. $x^2 - 10x + 25 = x^2 - 2(x)(5) + 5^2 = (x - 5)^2$

b. $16x^2 + 24x + 9 = (4x)^2 + 2(4x)(3) + 3^2 = (4x + 3)^2$

CHECKPOINT Now try Exercise 115.

The next two formulas show the sums and differences of cubes. Pay special attention to the signs of the terms.

Like signs Like signs

$$u^3 + v^3 = (u + v)(u^2 - uv + v^2) \qquad u^3 - v^3 = (u - v)(u^2 + uv + v^2)$$

Unlike signs Unlike signs

Example 9 **Factoring the Difference of Cubes**

Factor $x^3 - 27$.

Solution

$x^3 - 27 = x^3 - 3^3$ Rewrite 27 as 3^3.

$= (x - 3)(x^2 + 3x + 9)$ Factor.

CHECKPOINT Now try Exercise 123.

Example 10 **Factoring the Sum of Cubes**

a. $y^3 + 8 = y^3 + 2^3$ Rewrite 8 as 2^3.

$= (y + 2)(y^2 - 2y + 4)$ Factor.

b. $3(x^3 + 64) = 3(x^3 + 4^3)$ Rewrite 64 as 4^3.

$= 3(x + 4)(x^2 - 4x + 16)$ Factor.

CHECKPOINT Now try Exercise 125.

Trinomials with Binomial Factors

To factor a trinomial of the form $ax^2 + bx + c$, use the following pattern.

Factors of a

$$ax^2 + bx + c = (\boxed{}\,x + \boxed{})(\boxed{}\,x + \boxed{})$$

Factors of c

The goal is to find a combination of factors of a and c such that the outer and inner products add up to the middle term bx. For instance, in the trinomial $6x^2 + 17x + 5$, you can write all possible factorizations and determine which one has outer and inner products that add up to $17x$.

$$(6x + 5)(x + 1),\ (6x + 1)(x + 5),\ (2x + 1)(3x + 5),\ (2x + 5)(3x + 1)$$

You can see that $(2x + 5)(3x + 1)$ is the correct factorization because the outer (O) and inner (I) products add up to $17x$.

$$(2x + 5)(3x + 1) = \underset{\text{F}}{6x^2} + \underset{\text{O}}{2x} + \underset{\text{I}}{15x} + \underset{\text{L}}{5} = 6x^2 + \underset{\text{O + I}}{17x} + 5.$$

Example 11 Factoring a Trinomial: Leading Coefficient Is 1

Factor $x^2 - 7x + 12$.

Solution

The possible factorizations are

$$(x - 2)(x - 6), \quad (x - 1)(x - 12), \quad \text{and} \quad (x - 3)(x - 4).$$

Testing the middle term, you will find the correct factorization to be

$$x^2 - 7x + 12 = (x - 3)(x - 4).$$

CHECKPOINT Now try Exercise 131.

Example 12 Factoring a Trinomial: Leading Coefficient Is Not 1

Factor $2x^2 + x - 15$.

Solution

The eight possible factorizations are as follows.

$$(2x - 1)(x + 15) \qquad (2x + 1)(x - 15)$$
$$(2x - 3)(x + 5) \qquad (2x + 3)(x - 5)$$
$$(2x - 5)(x + 3) \qquad (2x + 5)(x - 3)$$
$$(2x - 15)(x + 1) \qquad (2x + 15)(x - 1)$$

Testing the middle term, you will find the correct factorization to be

$$2x^2 + x - 15 = (2x - 5)(x + 3). \qquad \text{O + I} = 6x - 5x = x$$

CHECKPOINT Now try Exercise 139.

STUDY TIP

Factoring a trinomial can involve trial and error. However, once you have produced the factored form, it is an easy matter to check your answer. For instance, you can verify the factorization in Example 11 by multiplying out the expression $(x - 3)(x - 4)$ to see that you obtain the original trinomial, $x^2 - 7x + 12$.

Factoring by Grouping

Sometimes polynomials with more than three terms can be factored by a method called **factoring by grouping.** It is not always obvious which terms to group, and sometimes several different groupings will work.

Example 13 Factoring by Grouping

Use factoring by grouping to factor $x^3 - 2x^2 - 3x + 6$.

Solution

$$\begin{aligned} x^3 - 2x^2 - 3x + 6 &= (x^3 - 2x^2) - (3x - 6) && \text{Group terms.} \\ &= x^2(x - 2) - 3(x - 2) && \text{Factor each group.} \\ &= (x - 2)(x^2 - 3) && \text{Distributive Property} \end{aligned}$$

CHECKPOINT Now try Exercise 147.

> **STUDY TIP**
>
> Another way to factor the polynomial in Example 13 is to group the terms as follows.
>
> $$\begin{aligned} &x^3 - 2x^2 - 3x + 6 \\ &= (x^3 - 3x) - (2x^2 - 6) \\ &= x(x^2 - 3) - 2(x^2 - 3) \\ &= (x^2 - 3)(x - 2) \end{aligned}$$
>
> As you can see, you obtain the same result as in Example 13.

Factoring a trinomial can involve quite a bit of trial and error. Some of this trial and error can be lessened by using factoring by grouping. The key to this method of factoring is knowing how to rewrite the middle term. In general, to factor a trinomial $ax^2 + bx + c$ by grouping, choose factors of the product ac that add up to b and use these factors to rewrite the middle term. This technique is illustrated in Example 14.

Example 14 Factoring a Trinomial by Grouping

Use factoring by grouping to factor $2x^2 + 5x - 3$.

Solution

In the trinomial $2x^2 + 5x - 3$, $a = 2$ and $c = -3$, which implies that the product ac is -6. Now, -6 factors as $(6)(-1)$ and $6 - 1 = 5 = b$. So, you can rewrite the middle term as $5x = 6x - x$. This produces the following.

$$\begin{aligned} 2x^2 + 5x - 3 &= 2x^2 + 6x - x - 3 && \text{Rewrite middle term.} \\ &= (2x^2 + 6x) - (x + 3) && \text{Group terms.} \\ &= 2x(x + 3) - (x + 3) && \text{Factor groups.} \\ &= (x + 3)(2x - 1) && \text{Distributive Property} \end{aligned}$$

So, the trinomial factors as $2x^2 + 5x - 3 = (x + 3)(2x - 1)$.

CHECKPOINT Now try Exercise 153.

Guidelines for Factoring Polynomials

1. Factor out any common factors using the Distributive Property.
2. Factor according to one of the special polynomial forms.
3. Factor as $ax^2 + bx + c = (mx + r)(nx + s)$.
4. Factor by grouping.

A.3 Exercises

VOCABULARY CHECK: Fill in the blanks.

1. For the polynomial $a_nx^n + a_{n-1}x^{n-1} + \cdot \cdot \cdot + a_1x + a_0$, the degree is ________, the leading coefficient is ________, and the constant term is ________.
2. A polynomial in x in standard form is written with ________ powers of x.
3. A polynomial with one term is called a ________, while a polynomial with two terms is called a ________, and a polynomial with three terms is called a ________.
4. To add or subtract polynomials, add or subtract the ________ ________ by adding their coefficients.
5. The letters in "FOIL" stand for the following.
   F ________ O ________ I ________ L ________
6. The process of writing a polynomial as a product is called ________.
7. A polynomial is ________ ________ when each of its factors is prime.

In Exercises 1–6, match the polynomial with its description. [The polynomials are labeled (a), (b), (c), (d), (e), and (f).]

(a) $3x^2$ (b) $1 - 2x^3$
(c) $x^3 + 3x^2 + 3x + 1$ (d) 12
(e) $-3x^5 + 2x^3 + x$ (f) $\frac{2}{3}x^4 + x^2 + 10$

1. A polynomial of degree 0
2. A trinomial of degree 5
3. A binomial with leading coefficient -2
4. A monomial of positive degree
5. A trinomial with leading coefficient $\frac{2}{3}$
6. A third-degree polynomial with leading coefficient 1

In Exercises 7–10, write a polynomial that fits the description. (There are many correct answers.)

7. A third-degree polynomial with leading coefficient -2
8. A fifth-degree polynomial with leading coefficient 6
9. A fourth-degree binomial with a negative leading coefficient
10. A third-degree binomial with an even leading coefficient

In Exercises 11–22, (a) write the polynomial in standard form, (b) identify the degree and leading coefficient of the polynomial, and (c) state whether the polynomial is a monomial, a binomial, or a trinomial.

11. $14x - \frac{1}{2}x^5$
12. $2x^2 - x + 1$
13. $-3x^4 + 2x^2 - 5$
14. $7x$
15. $x^5 - 1$
16. $-y + 25y^2 + 1$
17. 3
18. $t^2 + 9$
19. $1 + 6x^4 - 4x^5$
20. $3 + 2x$
21. $4x^3y$
22. $-x^5y + 2x^2y^2 + xy^4$

In Exercises 23–28, determine whether the expression is a polynomial. If so, write the polynomial in standard form.

23. $2x - 3x^3 + 8$
24. $2x^3 + x - 3x^{-1}$
25. $\dfrac{3x + 4}{x}$
26. $\dfrac{x^2 + 2x - 3}{2}$
27. $y^2 - y^4 + y^3$
28. $\sqrt{y^2 - y^4}$

In Exercises 29–46, perform the operation and write the result in standard form.

29. $(6x + 5) - (8x + 15)$
30. $(2x^2 + 1) - (x^2 - 2x + 1)$
31. $-(x^3 - 2) + (4x^3 - 2x)$
32. $-(5x^2 - 1) - (-3x^2 + 5)$
33. $(15x^2 - 6) - (-8.3x^3 - 14.7x^2 - 17)$
34. $(15.2x^4 - 18x - 19.1) - (13.9x^4 - 9.6x + 15)$
35. $5z - [3z - (10z + 8)]$
36. $(y^3 + 1) - [(y^2 + 1) + (3y - 7)]$
37. $3x(x^2 - 2x + 1)$
38. $y^2(4y^2 + 2y - 3)$
39. $-5z(3z - 1)$
40. $(-3x)(5x + 2)$
41. $(1 - x^3)(4x)$
42. $-4x(3 - x^3)$
43. $(2.5x^2 + 3)(3x)$
44. $(2 - 3.5y)(2y^3)$
45. $-4x\left(\frac{1}{8}x + 3\right)$
46. $2y\left(4 - \frac{7}{8}y\right)$

In Exercises 47–84, multiply or find the special product.

47. $(x + 3)(x + 4)$
48. $(x - 5)(x + 10)$
49. $(3x - 5)(2x + 1)$
50. $(7x - 2)(4x - 3)$
51. $(x^2 - x + 1)(x^2 + x + 1)$
52. $(x^2 + 3x - 2)(x^2 - 3x - 2)$
53. $(x + 10)(x - 10)$ **54.** $(2x + 3)(2x - 3)$
55. $(x + 2y)(x - 2y)$ **56.** $(2x + 3y)(2x - 3y)$
57. $(2x + 3)^2$ **58.** $(4x + 5)^2$
59. $(2x - 5y)^2$ **60.** $(5 - 8x)^2$
61. $(x + 1)^3$ **62.** $(x - 2)^3$
63. $(2x - y)^3$ **64.** $(3x + 2y)^3$
65. $(4x^3 - 3)^2$ **66.** $(8x + 3)^2$
67. $[(m - 3) + n][(m - 3) - n]$
68. $[(x + y) + 1][(x + y) - 1]$
69. $[(x - 3) + y]^2$ **70.** $[(x + 1) - y]^2$
71. $(2r^2 - 5)(2r^2 + 5)$ **72.** $(3a^3 - 4b^2)(3a^3 + 4b^2)$
73. $\left(\frac{1}{2}x - 3\right)^2$ **74.** $\left(\frac{2}{3}t + 5\right)^2$
75. $\left(\frac{1}{3}x - 2\right)\left(\frac{1}{3}x + 2\right)$ **76.** $\left(2x + \frac{1}{5}\right)\left(2x - \frac{1}{5}\right)$
77. $(1.2x + 3)^2$ **78.** $(1.5y - 3)^2$
79. $(1.5x - 4)(1.5x + 4)$ **80.** $(2.5y + 3)(2.5y - 3)$
81. $5x(x + 1) - 3x(x + 1)$
82. $(2x - 1)(x + 3) + 3(x + 3)$
83. $(u + 2)(u - 2)(u^2 + 4)$
84. $(x + y)(x - y)(x^2 + y^2)$

In Exercises 85–88, find the product. (The expressions are not polynomials, but the formulas can still be used.)

85. $\left(\sqrt{x} + \sqrt{y}\right)\left(\sqrt{x} - \sqrt{y}\right)$ **86.** $\left(5 + \sqrt{x}\right)\left(5 - \sqrt{x}\right)$
87. $\left(x - \sqrt{5}\right)^2$ **88.** $\left(x + \sqrt{3}\right)^2$

In Exercises 89–96, factor out the common factor.

89. $3x + 6$ **90.** $5y - 30$
91. $2x^3 - 6x$ **92.** $4x^3 - 6x^2 + 12x$
93. $x(x - 1) + 6(x - 1)$ **94.** $3x(x + 2) - 4(x + 2)$
95. $(x + 3)^2 - 4(x + 3)$ **96.** $(3x - 1)^2 + (3x - 1)$

In Exercises 97–102, find the greatest common factor such that the remaining factors have only integer coefficients.

97. $\frac{1}{2}x + 4$ **98.** $\frac{1}{3}y + 5$
99. $\frac{1}{2}x^3 + 2x^2 - 5x$ **100.** $\frac{1}{3}y^4 - 5y^2 + 2y$
101. $\frac{2}{3}x(x - 3) - 4(x - 3)$ **102.** $\frac{4}{5}y(y + 1) - 2(y + 1)$

In Exercises 103–112, completely factor the difference of two squares.

103. $x^2 - 81$
104. $x^2 - 49$
105. $32y^2 - 18$
106. $4 - 36y^2$
107. $16x^2 - \frac{1}{9}$
108. $\frac{4}{25}y^2 - 64$
109. $(x - 1)^2 - 4$ **110.** $25 - (z + 5)^2$
111. $9u^2 - 4v^2$ **112.** $25x^2 - 16y^2$

In Exercises 113–122, factor the perfect square trinomial.

113. $x^2 - 4x + 4$ **114.** $x^2 + 10x + 25$
115. $4t^2 + 4t + 1$ **116.** $9x^2 - 12x + 4$
117. $25y^2 - 10y + 1$ **118.** $36y^2 - 108y + 81$
119. $9u^2 + 24uv + 16v^2$ **120.** $4x^2 - 4xy + y^2$
121. $x^2 - \frac{4}{3}x + \frac{4}{9}$ **122.** $z^2 + z + \frac{1}{4}$

In Exercises 123–130, factor the sum or difference of cubes.

123. $x^3 - 8$ **124.** $x^3 - 27$
125. $y^3 + 64$ **126.** $z^3 + 125$
127. $8t^3 - 1$ **128.** $27x^3 + 8$
129. $u^3 + 27v^3$ **130.** $64x^3 - y^3$

In Exercises 131–144, factor the trinomial.

131. $x^2 + x - 2$ **132.** $x^2 + 5x + 6$
133. $s^2 - 5s + 6$ **134.** $t^2 - t - 6$
135. $20 - y - y^2$ **136.** $24 + 5z - z^2$
137. $x^2 - 30x + 200$ **138.** $x^2 - 13x + 42$
139. $3x^2 - 5x + 2$ **140.** $2x^2 - x - 1$
141. $5x^2 + 26x + 5$ **142.** $12x^2 + 7x + 1$
143. $-9z^2 + 3z + 2$ **144.** $-5u^2 - 13u + 6$

In Exercises 145–152, factor by grouping.

145. $x^3 - x^2 + 2x - 2$ **146.** $x^3 + 5x^2 - 5x - 25$
147. $2x^3 - x^2 - 6x + 3$ **148.** $5x^3 - 10x^2 + 3x - 6$
149. $6 + 2x - 3x^3 - x^4$ **150.** $x^5 + 2x^3 + x^2 + 2$
151. $6x^3 - 2x + 3x^2 - 1$ **152.** $8x^5 - 6x^2 + 12x^3 - 9$

In Exercises 153–158, factor the trinomial by grouping.

153. $3x^2 + 10x + 8$ **154.** $2x^2 + 9x + 9$
155. $6x^2 + x - 2$ **156.** $6x^2 - x - 15$
157. $15x^2 - 11x + 2$ **158.** $12x^2 - 13x + 1$

In Exercises 159–192, completely factor the expression.

159. $6x^2 - 54$

160. $12x^2 - 48$

161. $x^3 - 4x^2$

162. $x^3 - 9x$

163. $x^2 - 2x + 1$

164. $16 + 6x - x^2$

165. $1 - 4x + 4x^2$

166. $-9x^2 + 6x - 1$

167. $2x^2 + 4x - 2x^3$

168. $2y^3 - 7y^2 - 15y$

169. $9x^2 + 10x + 1$

170. $13x + 6 + 5x^2$

171. $\frac{1}{81}x^2 + \frac{2}{9}x - 8$

172. $\frac{1}{8}x^2 - \frac{1}{96}x - \frac{1}{16}$

173. $3x^3 + x^2 + 15x + 5$

174. $5 - x + 5x^2 - x^3$

175. $x^4 - 4x^3 + x^2 - 4x$

176. $3u - 2u^2 + 6 - u^3$

177. $\frac{1}{4}x^3 + 3x^2 + \frac{3}{4}x + 9$

178. $\frac{1}{5}x^3 + x^2 - x - 5$

179. $(t - 1)^2 - 49$

180. $(x^2 + 1)^2 - 4x^2$

181. $(x^2 + 8)^2 - 36x^2$

182. $2t^3 - 16$

183. $5x^3 + 40$

184. $4x(2x - 1) + (2x - 1)^2$

185. $5(3 - 4x)^2 - 8(3 - 4x)(5x - 1)$

186. $2(x + 1)(x - 3)^2 - 3(x + 1)^2(x - 3)$

187. $7(3x + 2)^2(1 - x)^2 + (3x + 2)(1 - x)^3$

188. $7x(2)(x^2 + 1)(2x) - (x^2 + 1)^2(7)$

189. $3(x - 2)^2(x + 1)^4 + (x - 2)^3(4)(x + 1)^3$

190. $2x(x - 5)^4 - x^2(4)(x - 5)^3$

191. $5(x^6 + 1)^4(6x^5)(3x + 2)^3 + 3(3x + 2)^2(3)(x^6 + 1)^5$

192. $\dfrac{x^2}{2}(x^2 + 1)^4 - (x^2 + 1)^5$

In Exercises 193–196, find all values of *b* for which the trinomial can be factored.

193. $x^2 + bx - 15$

194. $x^2 + bx + 50$

195. $x^2 + bx - 12$

196. $x^2 + bx + 24$

In Exercises 197–200, find two integer values of *c* such that the trinomial can be factored. (There are many correct answers.)

197. $2x^2 + 5x + c$

198. $3x^2 - 10x + c$

199. $3x^2 - x + c$

200. $2x^2 + 9x + c$

201. ***Cost, Revenue, and Profit*** An electronics manufacturer can produce and sell x radios per week. The total cost C (in dollars) of producing x radios is

$$C = 73x + 25{,}000$$

and the total revenue R (in dollars) is

$$R = 95x.$$

(a) Find the profit P in terms of x.

(b) Find the profit obtained by selling 5000 radios per week.

202. ***Cost, Revenue, and Profit*** An artisan can produce and sell x hats per month. The total cost C (in dollars) of producing x hats is

$$C = 460 + 12x$$

and the total revenue R (in dollars) is

$$R = 36x.$$

(a) Find the profit P in terms of x.

(b) Find the profit obtained by selling 42 hats per month.

203. ***Compound Interest*** After 2 years, an investment of \$500 compounded annually at an interest rate r will yield an amount of $500(1 + r)^2$.

(a) Write this polynomial in standard form.

(b) Use a calculator to evaluate the polynomial for the values of r shown in the table.

r	$2\frac{1}{2}\%$	3%	4%	$4\frac{1}{2}\%$	5%
$500(1 + r)^2$					

(c) What conclusion can you make from the table?

204. ***Compound Interest*** After 3 years, an investment of \$1200 compounded annually at an interest rate r will yield an amount of $1200(1 + r)^3$.

(a) Write this polynomial in standard form.

(b) Use a calculator to evaluate the polynomial for the values of r shown in the table.

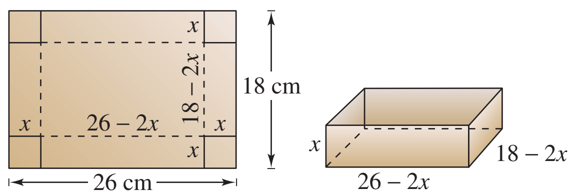
r	2%	3%	$3\frac{1}{2}\%$	4%	$4\frac{1}{2}\%$
$1200(1 + r)^3$					

(c) What conclusion can you make from the table?

205. ***Volume of a Box*** A take-out fast-food restaurant is constructing an open box by cutting squares from the corners of a piece of cardboard that is 18 centimeters by 26 centimeters (see figure). The edge of each cut-out square is x centimeters.

(a) Find the volume of the box in terms of x.

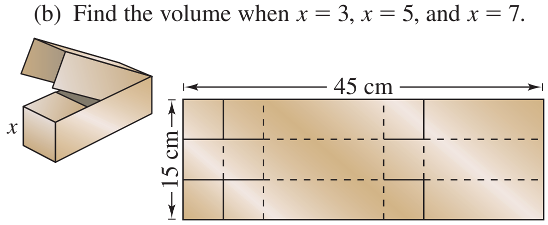
(b) Find the volume when $x = 1$, $x = 2$, and $x = 3$.



206. ***Volume of a Box*** An overnight shipping company is designing a closed box by cutting along the solid lines and folding along the broken lines on the rectangular piece of corrugated cardboard shown in the figure. The length and width of the rectangle are 45 centimeters and 15 centimeters, respectively.

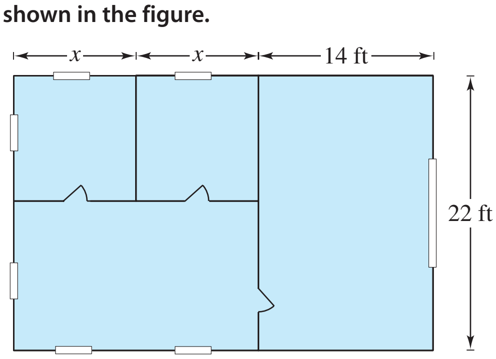
(a) Find the volume of the shipping box in terms of x.

(b) Find the volume when $x = 3$, $x = 5$, and $x = 7$.

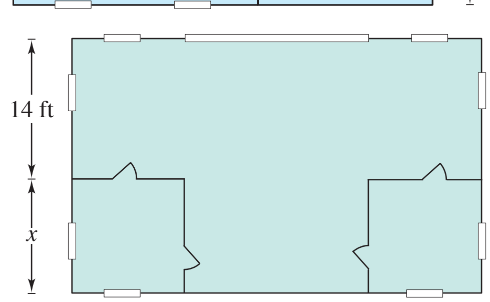


Geometry **In Exercises 207 and 208, find a polynomial that represents the total number of square feet for the floor plan shown in the figure.**

207.

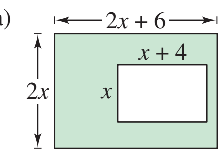


208.

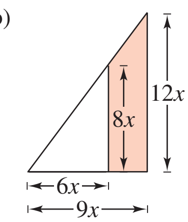


209. ***Geometry*** Find the area of the shaded region in each figure. Write your result as a polynomial in standard form.

(a)



(b)



210. ***Stopping Distance*** The stopping distance of an automobile is the distance traveled during the driver's reaction time plus the distance traveled after the brakes are applied. In an experiment, these distances were measured (in feet) when the automobile was traveling at a speed of x miles per hour on dry, level pavement, as shown in the bar graph. The distance traveled during the reaction time R was

$R = 1.1x$

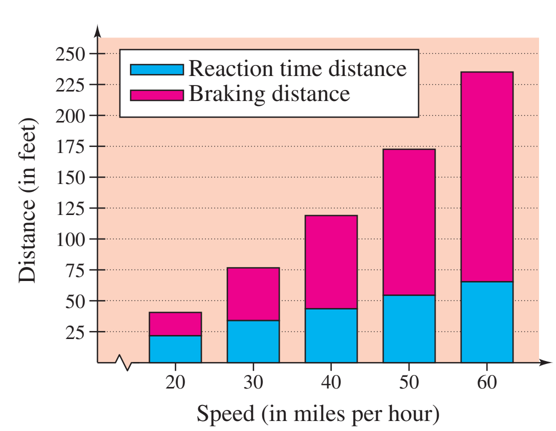
and the braking distance B was

$B = 0.0475x^2 - 0.001x + 0.23.$

(a) Determine the polynomial that represents the total stopping distance T.

(b) Use the result of part (a) to estimate the total stopping distance when $x = 30$, $x = 40$, and $x = 55$ miles per hour.

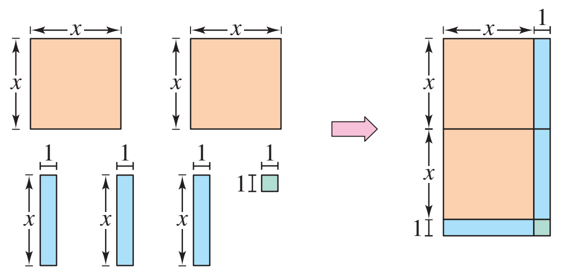
(c) Use the bar graph to make a statement about the total stopping distance required for increasing speeds.



Geometric Modeling **In Exercises 211–214, draw a "geometric factoring model" to represent the factorization. For instance, a factoring model for**

$$2x^2 + 3x + 1 = (2x + 1)(x + 1)$$

is shown in the figure.



211. $3x^2 + 7x + 2 = (3x + 1)(x + 2)$

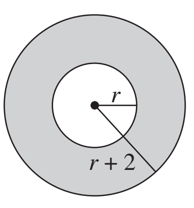
212. $x^2 + 4x + 3 = (x + 3)(x + 1)$

213. $2x^2 + 7x + 3 = (2x + 1)(x + 3)$

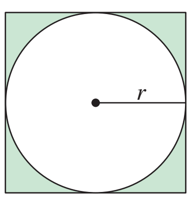
214. $x^2 + 3x + 2 = (x + 2)(x + 1)$

Geometry **In Exercises 215–218, write an expression in factored form for the area of the shaded portion of the figure.**

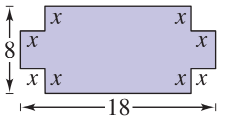
215.



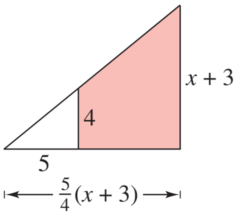
216.



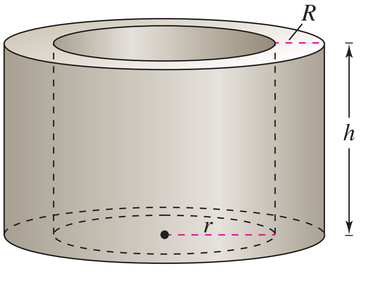
217.



218.



219. ***Geometry*** The volume V of concrete used to make the cylindrical concrete storage tank shown in the figure is $V = \pi R^2 h - \pi r^2 h$ where R is the outside radius, r is the inside radius, and h is the height of the storage tank.



(a) Factor the expression for the volume.

(b) From the result of part (a), show that the volume of concrete is

$$2\pi(\text{average radius})(\text{thickness of the tank})h.$$

220. ***Chemistry*** The rate of change of an autocatalytic chemical reaction is $kQx - kx^2$, where Q is the amount of the original substance, x is the amount of substance formed, and k is a constant of proportionality. Factor the expression.

Synthesis

True or False? **In Exercises 221–224, determine whether the statement is true or false. Justify your answer.**

221. The product of two binomials is always a second-degree polynomial.

222. The sum of two binomials is always a binomial.

223. The difference of two perfect squares can be factored as the product of conjugate pairs.

224. The sum of two perfect squares can be factored as the binomial sum squared.

225. Find the degree of the product of two polynomials of degrees m and n.

226. Find the degree of the sum of two polynomials of degrees m and n if $m < n$.

227. ***Think About It*** When the polynomial

$$-x^3 + 3x^2 + 2x - 1$$

is subtracted from an unknown polynomial, the difference is

$$5x^2 + 8.$$

If it is possible, find the unknown polynomial.

228. ***Logical Reasoning*** Verify that $(x + y)^2$ is not equal to $x^2 + y^2$ by letting $x = 3$ and $y = 4$ and evaluating both expressions. Are there any values of x and y for which $(x + y)^2 = x^2 + y^2$? Explain.

229. Factor $x^{2n} - y^{2n}$ completely.

230. Factor $x^{3n} + y^{3n}$ completely.

231. Factor $x^{3n} - y^{2n}$ completely.

232. ***Writing*** Explain what is meant when it is said that a polynomial is in factored form.

233. Give an example of a polynomial that is prime with respect to the integers.